Evanescent-mode propagation and quantum tunneling

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The tunneling of particles is in direct analogy with the transmission of evanescent electromagnetic waveguide modes as has been shown quite recently. We compare experimental data of an electromagnetic wave packet traversing an evanescent waveguide region with theoretical values derived for particle tunneling through a rectangular potential barrier. The transmission time was deduced by transformation of the experimental frequency data to the time domain. The data are in agreement and reveal superluminal wave-packet velocities for opaque evanescent regions.

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More than half a century has gone by since the introduction of the Schrödinger equation. The equation has given a description for the tunneling probability of wave packets; however, up to now there is no generally acknowledged method available for calculating a tunneling time, in spite of the importance of this quantity for modern microelectronic tunneling devices [1,2]. It has been pointed out that particle tunneling and the propagation of evanescent electromagnetic modes in a waveguide are formally similar; see, e.g., Refs. [3,4]. Previously, Martin and Landauer have emphasized that the propagation of an evanescent electromagnetic wave packet in a waveguide is equal to electron tunneling through a rectangular potential barrier [5]. The matching conditions imposed on the electric and magnetic fields for transverse electric, and for transverse magnetic waveguide modes at the interface between a propagating and an evanescent region are equivalent to those of the particle tunneling problem at a rectangular potential barrier. However, in spite of the formal similarities, one has to keep in mind the striking differences in the interpretation of a probability amplitude of a single quantum-mechanical process and of classical field amplitudes.

A short time after the tunneling experiment across thin insulating layers by Giaever [6] and others, Hartman studied tunneling of wave packets [7]. He calculated the tunneling time for a Gaussian wave packet crossing a rectangular barrier being given by the derivative of the phase delay to incident momentum. This is a so-called phase-time approach, which corresponds to the group velocity [1,7]. His phase-time approximation is based on a linear superposition of incident, reflected, and transmitted wave functions at the barrier boundaries. As shown in Fig. 1, the numerical results can be divided into three regions of barrier transition time depending on barrier thickness. For very thin barriers, the packet's transmission time is longer than the equal time, which represents the time for the incident packet to traverse a vacuum distance equal to the barrier thickness. For thicker, i.e., opaque barriers, the transmission time becomes independent of barrier thickness. For very thick barriers (not shown in Fig. 1), the transmitted wave packet is badly distorted, with the greatest contribution coming from Fourier components corresponding to energies just above the top of the barrier, where the transit time is approximately the equal time [7]. The intermediate case includes the possibility of superluminal particle velocity [2,5].

Recently, we have carried out experiments in order to investigate the propagation properties of evanescent electromagnetic modes in rectangular waveguides [8]. The wave number of the basic mode in a rectangular waveguide is given by the dispersion relation [3,4]

$$k^2 = (2\pi v/c)^2 - (2\pi v_e/c)^2,$$

(1)

![Graph of calculated particle transmission time](Image)

FIG. 1. Graphs of the calculated particle transmission time as a function of barrier thickness [7]. Where \( m \) is the particle mass, \( \hbar \) the Planck constant, and \( k/\epsilon \) is the incident wave number normalized to \( \epsilon \), the wave number equivalent to the potential barrier height. The dots represent the appropriately scaled experimental data of transmission time of evanescent electromagnetic waves [8]. Experimental parameters are as follows: center frequency of the Gaussian-like wave packet \( \nu = 8.7 \text{ GHz} \), \( \nu_{1} = 6.56 \text{ GHz} \), \( \nu_{2} = 9.49 \text{ GHz} \), \( a = 10, 40, 60, 80, \text{ and } 100 \text{ mm} \); for more details see Ref. [8].
where $\nu$ is the frequency, $c$ is the velocity of light, $\nu_c = c/2b$ is the cutoff frequency of the waveguide, and $b$ is the waveguide width (see Fig. 2). Thus, for $\nu \leq \nu_c$, the wave number is purely imaginary and this very mode is called an evanescent electromagnetic wave.

The transmission of an evanescent region was measured as a function of frequency, and with the length $a$ of the region as parameter [8]. The geometrical discontinuities at the waveguide transitions have a phase behavior similar to the ideal one-dimensional guide, having only a change in the refractive index, as discussed in Ref. [5]. For instance, at a frequency of 8.7 GHz, the phase shift calculated for the ideal waveguide is $-23^\circ$, whereas in the experiment with the geometrical discontinuities, $-15^\circ$ was measured [8], with similar deviations in the whole frequency interval. This deviation has a minor influence on the evaluated time-domain data. On the other hand, the transmission amplitude was always dominated by the attenuation of the cutoff section. Deviations from the ideal one-dimensional waveguide due to the geometrical discontinuities were not resolvable.

The time crossing the evanescent region was obtained from the frequency-domain-to-time-domain transform according to the relation

$$F'(t) = \int_{\nu_1}^{\nu_2} A(\nu)T(\nu)\exp(i2\pi\nu t) d\nu,$$

where $A(\nu)$ is the inverse Fourier transform of the initial Gaussian-like reference wave packet and $T(\nu)$ the frequency-dependent transmission function within the frequency range $(\nu_1, \nu_2)$. The experiment and its analysis are presented in Ref. [8]; here we want to emphasize that in the experiment, nearly ideal conditions are established: the stationary transmission and reflection coefficients at each frequency point are determined as if the source and the detector do not interfere with the evanescent region. This is realized by the process of calibration. From the electrotechnical point of view, this is a procedure to determine all systematic errors in the whole setup. This special procedure allows the evaluation of the scattering parameters of the cutoff waveguide section from measured data with a very good accuracy (better than $\pm 1^\circ$ in phase and $\pm 0.2$ dB in amplitude). From the physical point of view, this makes the source and the detector ideal and suppresses their interference with the cutoff waveguide; i.e., a perfectly matched device is equivalent to an infinitely long transmission line. If these error-corrected scattering data are transformed to the time domain via Eq. (2), the launching and detection of the corresponding wave packets happens without interference, too; i.e., the source and detector behave as if they have an infinite distance from the cutoff section.

The traversal times obtained by this transform—a procedure that works correctly for propagating waves with spatial oscillations in order to determine the time a signal travels along a transmission line or is reflected at a discontinuity of a circuit—equals Hartman’s theoretical data for particle tunneling as displayed in Fig. 1. For a short evanescent region $a \epsilon \leq 1$, the crossing time is longer than the equal time, and $\epsilon$ is the wave number equivalent to the barrier height of the evanescent region $\epsilon = \pi(1/b_2 - 1/b_1)$. For $a \epsilon \geq 1$ (opaque barrier), the crossing time becomes shorter than the equal time; the data are presented as dots in Fig. 1 for photon wave number corresponding to $0.7 \epsilon$. The signal velocity for crossing the evanescent region obtained from the relation $v = a/t$, with $t$ the crossing time and $a$ the region length, are shown in Fig. 2. In the same figure, experimental details of the evanescent-mode experiment are also given.

Hartman’s calculated phase time, as well as our experimental data, are essentially asymptotic in character, since they are derived as asymptotic characteristics for completed scattering events involving wave packets narrow in $k$ space [1]. (In our study, the frequency width of 0.5 GHz used corresponds to a packet width in $x$ space of 0.1 m, a value being shorter than the infinite source and detector distances to the evanescent waveguide.)

There are two remarkable results: (i) the frequency-domain-to-time-domain transform for electromagnetic modes yields traversal times that agree with Hartman’s wave-mechanical calculation for a particle, and (ii) with increasing length $a$ of the evanescent region, the group velocity extrapolated for this region exceeds the velocity of light. The traveling through an evanescent region appears to be done in zero time, a problem that was recently studied for particle tunneling by Low and Mende [9].

Summing up, evanescent electromagnetic waves propagate superluminally in opaque regions since the traversal time is independent of the region’s length [8]. The electromagnetic-mode experiment is assumed to correspond directly to particle tunneling. Obviously, Hartman’s model describes the tunneling of both particle and electromagnetic wave packets.
On causality proofs of superluminal barrier traversal of frequency band limited wave packets

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Abstract
Recently several approaches have been presented to prove causality for the measured superluminal photonic tunneling velocity. It is shown here that these proofs are not relevant for the frequency band limited microwave experiments in question (FM and AM signals) and that such experiments cannot be used to test Einstein causality.

1. Introduction
The analogy between quantum tunneling of particles and evanescent electromagnetic waves has been elaborated by Martin and Landauer recently [1]. The analogy was used by Enders and Nimtz to measure the tunneling time of frequency limited microwave packets in the evanescent regime of waveguides. The frequency limitation means that a central frequency carries an amplitude (AM) or frequency modulation (FM) within a distinct frequency band; this is for instance the technique used for radio broadcasting.

The microwave experiments have revealed superluminal velocities for the center of gravity and for the envelope of the amplitude modulated signals [2,3]. A front velocity of the signals was not determined. Any realistic signal is frequency limited and, consequently, has not a well defined front. Considering the particle aspect of a wave packet, the front velocity has no meaning, however, only this very velocity is relevant for the Einstein causality.

The tunneling experiments by Enders and Nimtz provoked a controversial discussion whether they are conflicting with causality [1,4–7]. Superluminal tunneling velocities on a single photon level have confirmed the previous microwave data [8].

First, we review the definitions of the different wave velocities. This will be followed by an introduction of the experimental procedure used to determine the group velocity of tunneling microwaves. After that we shall present arguments why proofs of causality, based on Gaussian signals, are not relevant for frequency band limited signals as used in the microwave experiments. As a result of this discussion Einstein causality cannot be tested in any realistic experiment due to inherent frequency band limitation.

2. Einstein causality and velocity terms
2.1. Einstein causality
Einstein causality is often described by the colloquial statement that nothing propagates faster than the speed of light in vacuum c. A definition of the velocity in question was given by Sommerfeld and Bril-
louin [9]: At the point \( x = 0 \) is a source, which is switched on at the time \( t = 0 \). Some distance \( d \) away from \( x \) no effect can be detected coming from \( x \) before the time \( d/c \). In almost any case the beginning of the signal is a discontinuity in the signal envelope or in a higher derivative. This front velocity \( v_F \) may not exceed \( c \) to fulfill Einstein causality.

The role of \( c \) as the limit of the front velocity of a wave packet is deduced from the Lorentz invariance of the Maxwell equations. The presence of boundary conditions, as in the case of reflection between two mirrors or in the case of an evanescent region without a wave solution, may break the Lorentz invariance according to Refs. [10–12].

2.2. Velocity terms

Other important terms of wave velocity are: phase, group, signal, energy, and particle velocity. Sommerfeld and Brillouin have based the definitions on a medium consisting of electrical charges with a Lorentz–Lorentz dispersion [9], which differs essentially from the dispersion of an evanescent medium (tunneling barrier). The latter has a purely imaginary wave number \( k \) and thus no wave solution.

The phase velocity \( v_{\phi} = \omega/k \) is the speed of a monochromatic wave, extended infinitely in space and time. \( v_{\phi} \) may become greater than \( c \). In consequence of the special dispersion, in the tunneling region \( v_{\phi} \) even becomes imaginary.

The group velocity \( v_{Gr} = d\omega/dk \bigg|_{\omega_0} \) is the speed of the center of gravity of a wave packet. At the resonance absorption of a dispersive medium \( v_{Gr} \) may become greater than \( c \) or even negative [9]. In a tunneling region \( v_{Gr} \) is imaginary like \( v_{\phi} \). As \( v_{Gr} \) also corresponds to the velocity of a particle by describing the particle as a wave packet, the interpretation of an imaginary velocity becomes physically unclear, causing much confusion [13].

The energy velocity in the case of electromagnetic waves is given e.g. by Ref. [14] as \( v_E = 2c\text{Re}(E \times H^*)/(E \cdot D^* + H \cdot B^*) \). In media characterized by a Lorentz–Lorentz dispersion \( v_E \) is real and below \( c \) for frequencies far away from resonance. Near resonance, where absorption takes place, the energy velocity is complex. A more general definition is given by Brillouin [15]: \( v_E = \text{Re}S/\text{Re}u \) with the Poynting vector \( S \) and the energy density \( u \).

Landauer has recently discussed the problem of an imaginary momentum which corresponds to an imaginary velocity and barrier traversal time [13], and to the reactive part of the energy flux.

The term signal velocity \( v_S \) was defined by Brillouin as the speed of the beginning of the essential part of the signal, which can be distinguished from the forerunners not only by a much higher amplitude but also by containing substantially lower frequencies than the forerunners. This implies that \( v_F \geq v_S \geq v_{Gr} \) holds for wave propagation [9].

The particle velocity \( v_p \) has to be defined in an experimentally accessible way: It is the quotient of time \( \Delta t \) between emission at a place \( x \) and “detection” at a distance \( d \) away from \( x \) and this distance, i.e. \( v_p = d/\Delta t \). In the case of photons the “whole” quantum will be absorbed in the detecting process. The emission time must be determined by a reference particle created at the same time and traveling another path freely to the detection device. Such a tunneling experiment at a single photon level was performed by Steinberg et al. [8]; they have reported \( v_p > c \) for the tunneling through a periodic dielectric hetero-structure in the photonic stopping band regime.

Finally the phase-time velocity \( d/(d\phi/d\omega) \) is presented, where \( d\phi/d\omega \) is the “phase time” or “group delay” that a wave packet needs to go through a region of finite length \( d \). The phase of a wave packet having the energy \( h\omega_0 \) has to be calculated from the trans-
mission function of that region with the argument $\omega_0$. The phase-time velocity is the real part of the group velocity.

In the definition of Einstein causality by Sommerfeld and Brillouin [9], the term information is not used. They are only discussing wave propagation which is characterized by a real component of $k$. The term information, however, is of great technical interest and will be discussed in Section 4.3.

3. Microwave experiments

Two different microwave experiments were performed: One in the frequency domain [2] and one in the time domain [3]. Both experiments were carried out with signals restricted to finite frequency bands. The waveguide structure under investigation was an undersized guide between normal-sized waveguides. The undersized part (the tunnel region) has a higher cutoff frequency than the adjacent parts.

3.1. Frequency domain

The tunneling time was determined from signals constructed from measurements in the frequency domain: For each monochromatic wave with frequency $\nu$ the complex transmission function $T(\nu)$ (amplitude and phase) of the barrier was measured. This was done in a defined frequency window $\nu_1 < \nu < \nu_2$, so that the signals were composed of evanescent modes only. The wave packet $A(\nu)$ was given by the Kaiser-Bessel function [16] inside the window and by zero outside. The peak of the Kaiser-Bessel function corresponds to the carrier frequency underlying the signal in the time domain. The Fourier transform

$$F(t) = \int_{\nu_1}^{\nu_2} d\nu A(\nu)T(\nu)e^{2\pi i\nu t}$$

yields the time response of the measured regions. As this signal is frequency band limited, it extends from $-\infty$ to $+\infty$ in the time domain. Since there is no defined front, such a signal cannot be used to check Einstein causality.

The rising edges of signals (which do not correspond to an ideal front) that have tunneled a distance of 60 mm were compared with those of signals detected at the front of the barrier attenuated to the same amount [3]. There was no significant time delay in the envelope of the tunneled signal, within the resolution of 10 ps, in spite of the additional path through the tunnel region of 60 mm length corresponding to a vacuum delay time of 200 ps.

4. Causality proofs

4.1. Front of a signal

As explained in Section 1, the Einstein causality requires the front velocity of a signal. Nitsch (see citation in Ref. [6]) has shown that this front velocity equals $c$ by analyzing the response function

$$F_{\Delta t}(\varphi) = \int_{-\infty}^{\infty} d\omega T(\omega)e^{-i\omega \varphi},$$

(1)
with a comparative transmission function \( T(\omega) = e^{i\omega \Delta L} \) with the increase \( \Delta L \) of the barrier length; this expression does not take the effects at the boundaries of the barrier into account. In the case of microwave tunneling through a region with lower dielectric constant, the dispersion relation \( k(\omega) = (\sqrt{\epsilon_{\mu}}/c) \times \sqrt{\omega^2 - \omega_c^2} \) with the cutoff frequency \( \omega_c = k_c/(\sqrt{\epsilon_{\mu}}/c) \) of the waveguide must be applied, where the wavenumber \( k_c \) depends only on the waveguide mode and geometry. By moving the path of integration into the complex \( \omega \)-plane, Hass [17,6] found

\[
F_{\Delta L}(\hat{\omega}) = 0 \quad \text{for} \quad \hat{\omega} < \frac{\Delta L}{c}.
\]  (2)

Let the signal in front of the barrier be \( \psi_0(t) \). Then the response behind the barrier is

\[
\psi_{\Delta L}(t) = \int_{-\infty}^{\infty} \, dt \, F_{\Delta L}(t-\tau) \psi_0(\tau). \]  (3)

Now let \( \psi_0(t) = 0 \) for \( t < 0 \), which defines a front at \( t = 0 \). Considering Eq. (2), one gets from Eq. (3)

\[
\psi_{\Delta L}(t) = \int_{0}^{t-\Delta L/c} \, d\tau \, F_{\Delta L}(t-\tau) \psi_0(\tau). \]  (4)

Thus \( \psi_{\Delta L}(t) = 0 \) for \( t < \Delta L/c \), which proves Einstein causality for the front of a signal. Furthermore, Hass and Busch [6] have stated that, taking into account the boundary effects, i.e. applying the full transmission function determined in Ref. [1], one would get the same result.

However, there is no experimental condition known by which such a well defined front could be generated. That makes the proof irrelevant for experiments.

### 4.2. Gaussian signal

Tunneling through barriers has been theoretically studied usually with Gaussian wave packets [1,4-6,18]. A priori, this approach cannot be applied to prove causality, as a Gaussian function represents an untemerated signal without a front.

Hass [17] studied a Gaussian \( A_0(\omega) \) in the frequency domain with a small variance so that the dispersion relation \( k(\omega) \) may be expanded into a Taylor series. Then one can write

\[
\psi_{\Delta L}(t) = \int_{-\infty}^{\infty} \, d\omega \, T(\omega) A_0(\omega) e^{i\omega t}
\]

\[
= C \int_{-\infty}^{\infty} \, d\omega \, A_{\Delta L}(\omega) e^{i\omega t},
\]

where \( A_{\Delta L}(\omega) \) is again a Gaussian for the tunneled signal. Compared to the original signal \( A(\omega) \) the tunneled signal is reshaped in consequence of the tunneling dispersion relation: The peak frequency is shifted to higher values by the amount \( \Delta \omega \) and the variance is increased.

Hass and Busch [6] have tried not to violate Einstein causality by arguing that due to pulse reshaping the incoming and the outgoing pulses are not causally connected. This naive argument holds neither for frequency-band limited signals nor in the case of a photonic stopping band having a symmetrical dispersion relation [19]. Nevertheless, as explained in Section 2.1 only the front velocity has to obey Einstein causality. So any argumentation for other parts of the signal is unnecessary; they may appear to break Einstein causality. Of course this statement is valid too in the case of the microwave experiments described in Section 3.

### 4.3. Technical signal

In electric engineering information is deduced from the amplitude modulation (AM) or from the frequency modulation (FM) of a carrier. That is how information is practically defined. According to this definition superluminal propagation of information (i.e. frequency limited signals) is possible in evanescent (tunnel) regions [20]. It may not be put into practice, because of the high attenuation caused by the barrier transition.

### 5. Conclusion

The microwave signals and any other signal used in frequency or time domain experiments are frequency band limited and therefore have no front as demanded
for a test of Einstein causality. A causality proof via the front of a signal or a pulse reshaping as presented in Ref. [6] is not relevant for the microwave experiments. Another important point in analyzing the experimental results is, that all the velocity terms discussed in the literature are based on an interaction mechanism of electromagnetic waves and bounded charges. This is described by a Lorentz–Lorentz dispersion, which differs essentially from the dispersion of an evanescent region being only determined by the boundary conditions. The first one holds for waves which have always a real wavenumber component, whereas the last one has a purely imaginary one, which does not correspond to a wave. This property theoretically results in an imaginary tunneling time and velocity [13].

The signals considered in the microwave experiments were unlimited in time and not Gaussian. Therefore Enders and Nimtz have never claimed that the front of a signal has travelled at superluminal speed [2,3]. However, they have stated that the peak and the rising edge of a frequency band limited wave packet propagate faster than c through a barrier. This result corresponds to a superluminal group and signal velocity and it was quite recently used to transmit Mozart’s Symphony No. 40 through a tunnel of 114 mm length at a speed of 4.7c [20].

In Section 4.3 we have introduced the term technical information on a basis of measurable quantities: maximum or center of gravity of the signal and rising or falling edges. In electrical engineering, information is generally transmitted with frequency limited signals by amplitude (AM) or frequency (FM) modulation of a carrier. In this sense it is justified to speak of a superluminal propagation of information through a barrier.

From the experimental point of view, it is not possible to violate Einstein causality: A front cannot be generated due to the frequency band limitation in a real experiment. However, it was shown in several experiments that wave packets and also single photons tunnel at a superluminal velocity.

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