A Full-wave Numerical Approach for Leaky Modes in EBG Structures

Vakhtang Jandieri¹, Paolo Baccarelli²,³, Guido Valerio⁴, and Giuseppe Schettini²,³

¹General and Theoretical Electrical Engineering (ATE), Faculty of Engineering
University of Duisburg-Essen, CENIDE — Center for Nanointegration Duisburg-Essen
Duisburg D-47048, Germany
²EMLAB3 Laboratory of Electromagnetic Fields, Department of Engineering, “Roma Tre” University, Italy
³CNIT, “Roma Tre” Unit, Via Vito Volterra 62, Rome 00146, Italy
⁴Laboratoire d’Electronique et Electromagnetisme, Sorbonne Université, Paris 75252, France

Abstract — During the last decade, an extensive research effort has been aimed at studying and developing electromagnetic bandgap (EBG) structures [1]. EBGs are artificial periodic dielectric or metallic structures in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. This frequency range is called the bandgap, which is analogous to the energy bandgaps for electrons in semiconductors [2]. A periodic array of infinitely long parallel cylinders is a typical kind of discrete periodic system. EBG structures composed of cylindrical inclusions, enclosed in a finite number of stacked layers, have inspired great interest because of their novel scientific and engineering applications as narrow-band filters, guiding devices, and substrates/covers for antennas.

EBG waveguides can be designed by removing one or more rows of the rods thus allowing to guide the electromagnetic waves with relatively low losses along particular directions [1]. When the number of the layers of cylinders is decreased, the EBG structure loses its complete band-gap properties and the modal field of the waveguide leaks out from the guiding structure. The losses along the structure are then described by a complex wavenumber. Knowledge of the real and complex propagation wavenumbers of bound and leaky modes supported by 2D EBG waveguides is essential for understanding of the fundamental parameters governing the design of leaky-wave antennas and of a variety of microwave and optical guiding devices. Therefore, the effective and rigorous full-wave modal analysis of EBG waveguides composed by multilayered arrays of 2D cylindrical inclusions is of particular interest.

In this regard, a rigorous and efficient full-wave numerical approach devoted to the modal analysis of 2D EBG waveguides is presented [3]. The proposed technique allows for the numerical study of bound and leaky modes propagating in various types of periodic and EBG structures. The method adopted here is based on the transition matrix (T-matrix) method, the Lattice Sums (LSs) and the generalized reflection and transmission matrices characterizing the nature of the EBG structure. Recently developed fast and accurate calculation for the LSs, based on higher-order spectral and spatial Ewald representations that are highly convergent also in the case of complex propagation wavenumbers [3, 4], is used. The proposed approach has shown a Gaussian convergence and allows for the correct spectral determination of each spatial harmonic constituting the leaky modal field.

We have numerically tested the accuracy and efficiency of the method for several types of periodic and EBG structures, including conventional dielectric periodic and EBG waveguides as well as most challenging plasmonic chains, where the intrinsic losses of the scatterers are properly considered. An excellent agreement has been observed in all cases. Future developments concern the application of the proposed method to the analysis and design of EBG based leaky-wave antennas, where the radiative features can be explained in terms of suitable leaky modes propagating along the EBG waveguides [5]. Radiation patterns both in infinite and truncated EBG structures are under investigation.

REFERENCES


A Full-Wave Numerical Approach for Leaky Modes in EBG Structures

V. Jandieri ¹, P. Baccarelli ², G. Valerio ³ and G. Schettini ²

¹ General and Theoretical Electrical Engineering (ATE), Faculty of Engineering, University of Duisburg-Essen, D-47048, Duisburg, Germany.
² Roma Tre University, Department of Engineering, Via Vito Volterra 62, 00146 Rome, Italy.
³ Sorbonne Université, Laboratoire d'Electronique et Electromagnetisme, 75252, Paris, France.

Motivation (1)

2D Electromagnetic-Band-Gap (EBG) Structure

Periodic chain

Planar 2D waveguide structure

The modal field is represented as a superposition of an infinite number of space harmonics

Goal: Derivation of the complex wavenumbers for bound modes in their stop-band regimes and leaky modes in their physical and non-physical regions

Space-harmonic complex wavenumber

$$k_{xn} = \beta_n + i\alpha, \quad n = 0, \pm 1, \pm 2, ...$$

$$\beta_n = \beta_0 + \frac{2\pi n}{h}$$

Modal attenuation (leakage) constant

Space-harmonic phase constant
Motivation (2)

- The complex wavenumbers are found by applying a rigorous and efficient formulation based on the Lattice Sums (LSs) technique combined with the Transition-matrix (T-matrix) approach and the recursive algorithm for the multilayered structure [1].
- The method is highly efficient, since the LSs are evaluated by using an effective Ewald approach [2] and a recursive relation for the layered structure is based on a simple matrix multiplication.
- The method allows for the appropriate choice of the spectral determination for each space harmonic in order to consider both proper and improper modal solutions [2].
- Radiative features of EBG Fabry-Perot cavities excited by simple localized sources (line or Hertzian dipole sources) at microwave and millimeter waves can be explained in terms of the leaky modes supported by the relevant open waveguide [3].

Formulation of the Problem (1)

Single array

\[ e^{i k_{x0} x} \rightarrow e^{i k_{xn} x} , \quad k_{xn} = k_{x0} + \frac{2n\pi}{h} \]

Reflected fields \((y > 0)\)

\[ \psi^r_n (x, y) = r_n^{(+)} a^i e^{i(k_{xn}x + k_{yn}y)} , \quad r_n^{(+)} = u_n^{(+)}T \cdot (I - T \cdot L)^{-1} \cdot T \cdot p^- \]

\(r_n^{(+)}: 0\)-th incident wave \(\rightarrow n\)-th reflected wave

T-matrix is obtained in a closed form for cylindrical inclusions. It is a diagonal matrix.
Formulation of the Problem (2)

Single array

\[ e^{ik_{x0}x} \rightarrow y \]
\[ e^{ik_{xn}x} \rightarrow x \]

Transmitted fields \((y<0)\)

\[ \psi_n^T(x, y) = f_n^{(-)} a^i e^{i(k_{xn}-k_{yn}y)} , \quad f_n^{(-)} = \delta_{n0} + u_n^{(-)T} \cdot (I - T \cdot L)^{-1} \cdot T \cdot p^* \]

\( f_n^{(-)} \): 0-th incident wave \( \rightarrow \) \( n \)-th transmitted wave

\[ u_n^{(\pm)} = [u_{nm}^{(\pm)}] = \left[ \frac{2(-i)^m}{k_{yn}h} e^{\pm ima_n} \right], \]

\[ \alpha_n = \cos^{-1} \left( \frac{k_{xn}}{k_0} \right), \quad k_{yn} = \sqrt{k^2 - k_{xn}^2} \]

The proper or improper features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber \( k_{yn} \).

Formulation of the Problem (3)

\( R^{(\pm)} \): Reflection matrix for downgoing and upgoing space harmonics

\( F^{(\pm)} \): Transmission matrix for downgoing and upgoing space harmonics
Lattice-Sum (LS)

\[ L_m(k_h, k_{x0}h) = \sum_{n=1}^{\infty} H^1_m(nkh)[e^{ik_{x0}hn} + (-1)^m e^{-ik_{x0}hn}] \]

\( L_m \) depends only on \( k, h \) (i.e., the period), and \( k_{x0} \) (i.e., the fundamental space-harmonic wavenumber); It is independent of the polarization of the incident field. It uniquely characterizes a periodic array of sources.

Real wavenumber \([1, 2]\): Slow converging series

\[ k_{x0} = \beta_0 \]

Complex wavenumber \([3, 4]\): Exponentially diverging series

\[ k_{x0} = \beta_0 + i\alpha \]


LSs for the complex wavenumber can be accurately calculated using Ewald method. We calculate separately spectral and spatial series \([3, 4]\):

\[ L_m = L^\text{spectral}_m (kh, k_{x0}h) + L^\text{spatial}_m (kh, k_{x0}h) \]

Lattice-Sum Ewald Spectral Series

After several mathematical manipulations, for the Spectral series we finally obtain:

\[ L^\text{spectral}_m (kh, k_{x0}h) = \frac{2i}{h} \sum_{m=0}^{\infty} \left( \frac{k_m}{k} \right)^m \sum_{q=0}^{[m/2]} (-1)^q \left( \frac{m}{2q} \right) \left( \frac{k_m}{k_{x0}} \right)^{2q} C_{q,n}, \quad m \geq 0 \]

\[ C_{q,n} = \frac{1}{k_{yn}} \text{erfc} \left( -i \frac{h k_{yn}}{2 E_{spl}} \right) - \left( \frac{h k_{yn}}{2 E_{spl}} \right)^2 \sum_{s=1}^{q} \frac{(-i \frac{h k_{yn}}{2 E_{spl}})^{1-2s}}{k_{yn} \Gamma \left( \frac{3}{2} - s \right)} \]

\[ \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \]

The \textbf{spectral} higher-order Ewald series presents a \textbf{very fast Gaussian convergence} also for complex \( k_{x0} \)

The \textbf{proper} or \textbf{improper} features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber \( k_{yn} \).
Lattice-Sum Ewald Spatial Series (1)

For the Spatial series, we obtain:

\[
I_{m}^{\text{Spatial}}(kh,k_{0}h) = \delta_{m,0} \left[ -1 - \frac{i}{\pi} \text{Ei} \left( \frac{k^{2}h^{2}}{4E_{\text{spl}}^{2}} \right) \right] + \frac{2^{m+1}}{i\pi} \sum_{n=1}^{\infty} \left[ e^{ink_{0}h} + (-1)^{m} e^{-ink_{0}h} \right] \left( \frac{n}{kh} \right)^{m} \int_{E_{\text{spl}}}^{\infty} e^{-n^{2}\eta^{2} \frac{k^{2}h^{2}}{4\eta^{2}}} d\eta, \quad m \geq 0,
\]

How to compute this integral? A numerical integration could be slow and not robust…

\[
I_{m} = \int_{E_{\text{spl}}}^{\infty} \frac{e^{-n^{2}\eta^{2} \frac{k^{2}h^{2}}{4\eta^{2}}}}{\eta^{2m+1}} d\eta, \quad m \geq 0,
\]

Lattice-Sum Ewald Spatial Series (2)

A recurrence relation in \( m \) have been obtained to significantly speed up the evaluation of the integrals in the spatial Ewald series

\[
I_{m+1} = \frac{1}{2n^{2}} \left( 2mI_{m} - \frac{k^{2}h^{2}}{2} I_{m-1} + E_{\text{spl}}^{2m} e^{-n^{2}E_{\text{spl}}^{2}} e^{k^{2}h^{2}/4E_{\text{spl}}^{2}} \right)
\]

\( I_{0} \) and \( I_{1} \) can be easily obtained as

\[
I_{0} = \frac{1}{2} \sum_{p=0}^{\infty} \left( \frac{kh}{2E_{\text{spl}}} \right)^{2p} \frac{1}{p!} E_{\text{spl}}^{p} \left( n^{2}E_{\text{spl}}^{2} \right) \quad I_{1} = \frac{E_{\text{spl}}^{2}}{2} \left[ \frac{1}{n^{2}E_{\text{spl}}^{2}} e^{-n^{2}E_{\text{spl}}^{2}} + \sum_{p=1}^{\infty} \left( \frac{kh}{2} \right)^{2p} \frac{1}{p!} E_{\text{spl}}^{p} \left( n^{2}E_{\text{spl}}^{2} \right) \right]
\]

The spatial higher-order Ewald series also presents a very fast Gaussian convergence also for complex \( k_{0}h \).
Spectral Properties of the Modal Solution

Behavior of each space harmonic in the air region, e.g.: $y > 0, \ e^{ik_{yn}y}$

$$k_{yn} = \sqrt{k^2 - k_{xn}^2} = \sqrt{k^2 - (\beta_n + i\alpha)^2} = \beta_{yn} + i\alpha_{yn}$$

Exponentially decaying

Proper determination: $\Im\{k_{yn}\} > 0$

Improper determination: $\Im\{k_{yn}\} < 0$

Physical choice for each space harmonic of the spectral series

<table>
<thead>
<tr>
<th>Surface Wave</th>
<th>Forward Wave</th>
<th>Backward Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>(proper)</td>
<td>(improper leaky wave)</td>
<td>(proper leaky wave)</td>
</tr>
<tr>
<td>Slow harmonics: $</td>
<td>\beta_n</td>
<td>&gt; k : \Im{k_{yn}} &gt; 0$</td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>Fast harmonics: $</td>
<td>\beta_n</td>
<td>&lt; k : \Im{k_{yn}} &lt; 0$</td>
</tr>
</tbody>
</table>

Full-Wave Modal Analysis. Numerical Results

Fourier Series Expansion Method (FSEM) Combined with Perfectly Matched Layers (PMLs)

- For validation purposes, a Fourier Series Expansion method (FSEM) with perfectly matched layers (PMLs) has been implemented to analyze 2-D EBG waveguides composed by cylindrical inclusions, whose section can have an arbitrary geometry.
- The electric and magnetic fields are approximated by truncated Fourier series.
- The FSEM uses the staircase approximation of the circular section by applying several multilayered thin rectangular strips.
- A substantial number of numerical tests are required to properly choose the PML parameters in order to distinguish the leakage loss from the material loss caused by the assumed conductivity in the PMLs.

W1 Type EBG Waveguide: Improper Leaky Mode (1)

The periodic array of dielectric cylindrical rods has a bandgap region in the normalized frequency range $0.303 < \frac{h}{\lambda_0} < 0.432$

Lowest order TE leaky mode $(E_z, H_x, H_y)$

The $n = 0$ space harmonic is fast and has an improper determination in the LST, whereas all other harmonics are proper

The results obtained with the LST and FSEM with PML are in very close agreement with an accuracy of at least four digits

W1 Type EBG Waveguide: Improper Leaky Mode (2)

When the number of the EBG layers is increasing the attenuation constant substantially decreases.

$n = 0$ space harmonic is fast and has an improper determination in the LST

The LST is more efficient than the FSEM with PML: 0.02 s against 20 s per one frequency point with the same 3.6 GHz Intel Core i7 with 8 GB RAM
Periodic Chain of Dielectric Circular Rods

Complete Brillouin diagram with the details of the backward and forward fast-wave regions for the $n = -1$ harmonic (proper/improper determinations), the closed and open stop-band regions, and the grating lobe due to the simultaneous radiation from the $n = -1$ and $n = -2$ harmonics.

**Brillouin Diagram**

Backward leaky wave: phase and group velocities of opposite signs
Forward leaky wave: phase and group velocities are in the same direction
Periodic Chain of *Plasmonic* Circular Rods

\[ \varepsilon = 1 - i \frac{\nu \omega_p^2}{\omega(1+i\nu\omega)} \]

TM mode \((H, E_x, E_y)\)

\[ \nu = 1.45 \times 10^{-14} \text{s} \]

\[ \omega_p = 1.32 \times 10^{16} \text{rad/s} \]

\[ r = 0.4167 \ h \]

Propagation constant \(\beta h/2\pi\) and attenuation constant \(\alpha h/2\pi\) versus the normalized frequency \(h/\lambda_0\) for the fundamental and higher-order mode of \(H\)-wave for the 2-D periodic chain composed of the silver circular rods having radius \(r = 0.4167 h\). Solid line represent the results obtained based on the present method and the circles represent the results shown in Ref. [1].


LW Radiation from infinite EBG structures

Normalised Radiation Patterns (Infinite EBG LWAs)

\(h = 7 \text{ mm} \)

\(\varepsilon_1 = 11.9\varepsilon_0 \)

\(r_1 = 0.2 \ h \)

\(N = 2 \)

12.92 GHz

13.07 GHz

<table>
<thead>
<tr>
<th>Broadside frequency</th>
<th>(f(\text{GHz}))</th>
<th>(\beta / k_0)</th>
<th>(\alpha / k_0)</th>
<th>(\theta_{\text{in}}(\text{C}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.92</td>
<td>0.1065</td>
<td>0.1066</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>13.07</td>
<td>0.1923</td>
<td>0.0531</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Perfect agreement between normalized radiation patterns:

Total Field excited by an electric line source vs. Leaky-Wave field
LWAs based on Truncated EBG Structures: Radiation at Broadside

Far-field features of **Truncated** EBG LWAs

- Theoretical LW Radiation Pattern (LWRP) based on the physical-optics approximation (i.e., perfect absorbers at the antenna truncations).
- Full-wave calculation of the field radiated by an elementary source in a truncated structures through an ad-hoc implementation of the Cylindrical Wave Approach (CWA).
- CST microwave studio: The excitation of a 2-D structure has been properly emulated.

The length $L/2$ (=38.5 mm) of the LWA has been fixed to radiate 90% of the injected power at the broadside frequency (12.92 GHz).

Excellent agreement between normalized radiation patterns obtained with the LW theory, CWA, and CST.

LWRP vs CWA

CWA vs CST

LWAs based on Truncated EBG Structures: 3dB Bandwidth at Broadside

Directivity Patterns

Directivity patterns obtained with the CWA and validated with CST show a fractional 3dB bandwidth at broadside a.e. to 10% for a truncated EBG LWA with a 90% radiation efficiency at broadside frequency.

Near-field patterns obtained with the CWA

12.92 GHz (Broadside frequency) 13.34 GHz 12.14 GHz

Directivity at Broadside ($\theta = 0^\circ$)
Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (1)

Layered crossed-arrays of cylindrical objects

Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (2)

Z-array:  $\mathbf{R}_z, \mathbf{F}_z$  $(E_z, \tilde{H}_z)$

X-array:  $\mathbf{R}_x, \mathbf{F}_x$  $(E_x, \tilde{H}_x)$

Coordinate transformation

Stacked

Crossed-array:  $\mathbf{R}_z^{(c)}, \mathbf{F}_z^{(c)}, \mathbf{R}_x^{(c)}, \mathbf{F}_x^{(c)}$

Layered crossed-arrays
Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (3)

\[
\begin{align*}
R_{v,v+1}^U &= R_x^{(+)} + F_x^{(+)} W R_{v+1,v+2}^U \Lambda_{v+1}^{(-)} W F_x^{(-)} \\
F_{1,Q+1}^\cap &= F_x^{(-)} \Lambda_0^{(-)} \Lambda_{Q-1}^{(-)} \cdots \Lambda_2^{(-)} W F_x^{(-)} \\
R_{v+1,v}^\cap &= R_x^{(-)} + F_x^{(-)} W R_{v,v-1}^\cap \Lambda_{v}^{(+)} W F_x^{(+)} \\
F_{Q+1,1}^U &= F_x^{(+)} \Lambda_2^{(+)} \Lambda_3^{(+)} \cdots \Lambda_{Q-1}^{(+)} W F_x^{(+)} \\
\Lambda_{v+1}^{(-)} &= \left[ I - W R_x^{(-)} W R_{v+1,v+2}^U \right]^{-1} \\
\Lambda_v^{(+)} &= \left[ I - W R_x^{(+)} W R_{v,v-1}^\cap \right]^{-1} \\
W &= \left[ e^{i \gamma_{(m)} d} \delta_{(l)l'} \right]
\end{align*}
\]

Conclusions

- A full-wave numerical approach for the analysis of modes with complex propagation wavenumber in periodic and bandgap structures composed of 2D cylindrical inclusions has been proposed.
- The method is based on the lattice sums (LSs) technique and has been suitably adapted to the analysis of modes with complex propagation wavenumbers, by applying higher-order Ewald representation, in terms of spectral and spatial series having Gaussian convergence.
- All the possible bound and leaky modes propagating along periodic and bandgap structures composed of 2D cylindrical inclusions can be considered.
- An exhaustive analysis of two reference 2D EBG waveguides has allowed us to characterize the relevant Pass-Band and Band-Gap Zones and the Radiative Regions.

Future works:

- Analysis of leakage and radiative phenomena in 2D EBG structures
- Design of filters and periodic Leaky Wave Antennas based on EBG structures
Thank You!