Subgridding capabilities of the Wavelet-transformed FDTD scheme

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Abstract

In this paper an investigation on computational efficiency in terms of performance and accuracy for a spatial wavelet-transformed FDTD scheme is presented. The method shows subgridding features with computation times even smaller compared to the classical FDTD method.

Introduction

Modern problem settings in the realm of e.g. microwave engineering are constantly growing in complexity, size and needs, but the resulting demand for more powerful computer resources is not an issue that has to be resolved solely by the chip manufacturers. Hence, new numerical or optimized methods become necessary for satisfying upcoming customer requirements. One of the most versatile methods in computational electromagnetics is the finite-difference time-domain (FDTD) method [1]. A subclass of FDTD is given by the multi-resolution time-domain (MRTD) methods [2]. A comprehensive evaluation of the MRTD method can be found in [3]. Another modification is the Wavelet-transformed FDTD (WT-FDTD) scheme [4]. Here, the classical FDTD operators are transformed into the wavelet domain using discrete and multidimensional wavelet operators in order to achieve subgridding capabilities. The mathematical core is completely different compared to MRTD, since MRTD approximates the field values through wavelet functions, whereas the WT-FDTD transforms the operator coefficients. In the paper a short introduction into the WT-FDTD method is given. To evaluate the subgridding capabilities, a L-shaped cavity and a double loaded resonator are analyzed, where the first one delivers usually inaccurate results [3] and the second one is not solvable with the MRTD method due to its underlying mathematical principle [5]. WT-FDTD shows better performance (i.e. mega cells per second) than FDTD using the same discretization setup or it shows better accuracies. Moreover it shows accuracies and processing times between coarse and finer resolution, where the latter adverts to the WT-FDTD's subgridding capabilities.

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Wavelet-transformed FDTD

The first integral part of the WT-FDTD scheme is the transformation of all spatial operators. The resulting property of the transformed operator's implements automatically transformed field components. For a more compact notation of a three dimensional (3D) environment, component and direction indices are numbered in modulo three sense ($E_x = E_0$, $E_y = E_1$ and $E_z = E_2$). Starting with Faraday's law of the classical FDTD scheme and applying the discrete wavelet operator W, we yield

$$\begin{aligned} H_1^{n+.5} &= H_1^{n-.5} - \frac{\Delta t}{\mu_1}\Big[\partial_2 E_0^n - \partial_0 E_2^n\Big] \\ \Rightarrow W \circledast H_1^{n+.5} &= W \circledast \Big(H_1^{n-.5} - \frac{\Delta t}{\mu_1}\Big[\partial_2 E_0^n - \partial_0 E_2^n\Big]\Big) \\ \tilde{H}_1^{n+.5} &= \tilde{H}_1^{n-.5} - W\,(\frac{\Delta t}{\mu_1})W^{-1}\,W\Big[\partial_2 E_0^n - \partial_0 E_2^n\Big] \\ &= \tilde{H}_1^{n-.5} - \tilde{M}_1\,W\,\partial_2\,W^{-1}\,W\,E_0^n - W\Big(\partial_0 E_2^n\Big) \\ &= \tilde{H}_1^{n-.5} - \tilde{M}_1\Big[\tilde{\partial}_2\,\tilde{E}_0^n - \tilde{\partial}_0 \tilde{E}_2^n\Big] \quad \text{with} \quad W \cdot W^{-1} = I \end{aligned} \tag{1}$$

Ampere's law can be transformed straightforward, hence equation (1) looks very familiar to the classical counterpart, except that it contains now tensorial operator maps of the material and derivation operators (indicated by the tilde) into the appropriate subfilter systems. The operator W and W^{1} are computed by a tensor product of one-dimensional and discrete transformation operators with the chosen wavelet filter. The underlying mathematical principle behind the WT-FDTD is based on the $W{\bullet}O{\bullet}W^{1}$ operator transformation, which expands through W the access of the original operator O into the subzones of the wavelet domain and concentrates through the W^{1} concatenation the access of the pre-state operator $W{\bullet}O$ into the appropriate subzones by operator coefficient reduction. The common case of homogenous material yields through $W{\bullet}O{\bullet}W^{1}$ into identity. This reduces the operator access into its own filter zone and omits additional overhead to other filter zones. Inhomogeneous material produces higher numerical costs, but could be seamless computed. On the one hand, it should be fairly quoted, that the WT-FDTD scheme easily tends to higher numerical cost without any essential negligence and on the other hand renders the linear transformation to inherit all advantages from the classical FDTD scheme, which includes also the well-known stability criterion. This behavior is in total disagreement with the MRTD scheme and it can be shown both, empirically and analytically that the stability criterion is supported even if the detail operators are neglected.

Analysis

The evaluation of the WT-FDTD is carried along the simulations of the TM_z eigenmodes of two resonator structures, which are enclosed by PECs. The L-shaped cavity, embedded in Fig.1, showed weaker accuracy with MRTD than with classical FDTD [3], whereas the double loaded resonator, displayed in Fig. 2, cannot be computed with the MRTD method, because dielectric perturbations are handled with the image principle [5] and the chosen cavity leads to infinite images. In order to achieve a fair comparison between WT-

FDTD and its classical counterpart, we optimized the codes equally, setting them up as single threaded applications and compiled them with the same compiler using identical optimization flags. As already mentioned before, the disregard of operators is crucial to achieve an efficient scheme. The analysis were carried out using the smallest crucial grid in the wavelet domain for a total elapsed time of 1 ms and excited with a Gaussian pulse from 0 to 300 MHz. The smallest possible wavelength λ_{min} is approximated by 1 m and the performance is based upon a PC with an Intel Core 2 Duo (Merom) at 2.4 GHz. Better performance (mega cells per second, execution time) was achieved at the expense of a slightly lower accuracy when increasing the resolution for all applications. The corresponding data for each resonator structure is depicted in Fig.1/Fig.3 and Fig.1/Fig.3, respectively. The fundamental mode of the L-shaped resonator has been retrieved from a mode-matching analysis [4], yielding a reference value f_{ref} = 76.049 MHz. The fundamental mode for the double loaded resonator was computed with an eigenvalue solver to f_{ref} = 35.155 MHz. For the L-shaped cavity the x-directed WT-FDTD schemes shows better accuracies than the classical FDTD counterpart, whereas the double loaded resonator has similar accuracy for the y-directed scheme. The common case, depicted by the double loaded cavity in Fig. 3, shows accuracies between a classical FDTD single and a double resolution scenario, which is anticipated as the WT-FDTD's subgridding capability.

Conclusion

The presented method has proven to be suitable for subgridding without any special time-step adjustments. Since the transformation of operator coefficients from the classical FDTD scheme is seamless, the inclusion of lossy or even dispersive materials [6] becomes straightforward. It is worth noting, that for three dimensional structures the speedup is expected to further increase, due to the higher compressions rates. Next steps planed for the future include the extension of the method to 3D problems shielded by absorbing boundaries.

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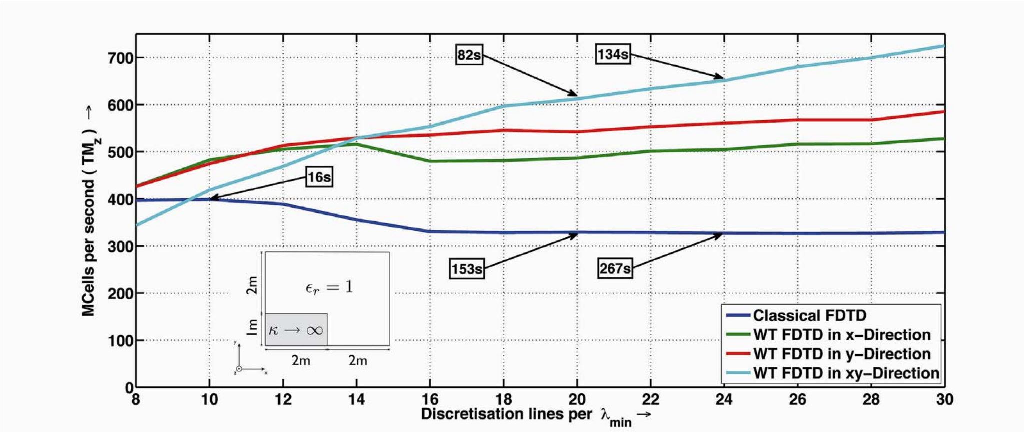

Figure 1: Performance analysis for the L-shaped resonator.

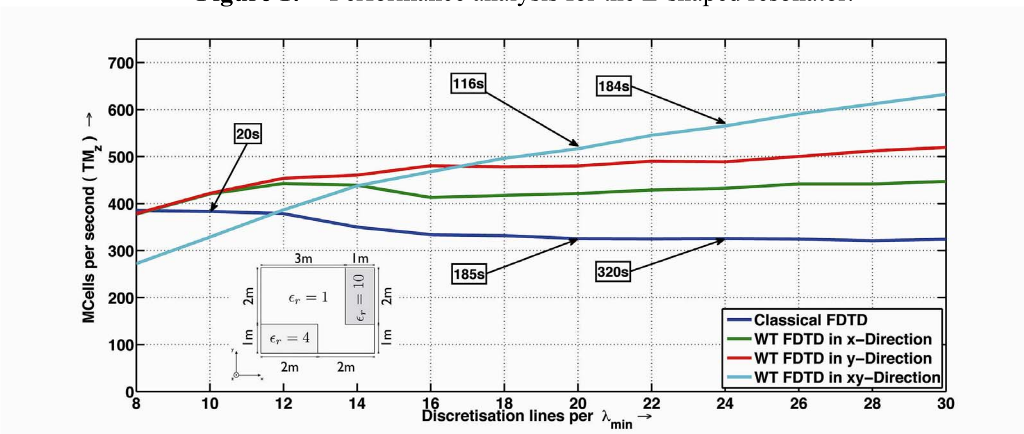

Figure 2: Performance analysis for the double loaded resonator.

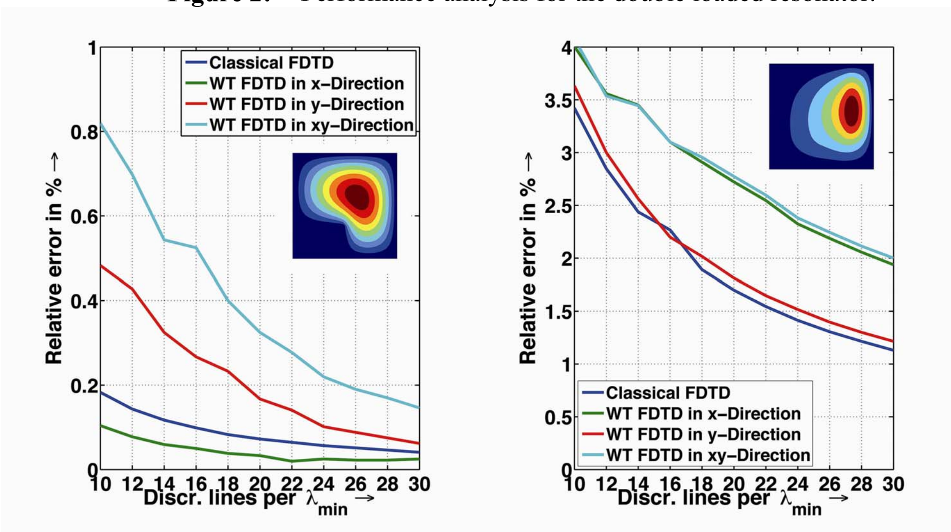

Figure 3: Convergence plot of the accuracy and mode pattern of the fundamental mode. L-shaped resonator is at the left, whereas the double loaded resonator is at the right.