Double-Lorentz Transmission Line Metamaterial and its Application to Tri-Band Devices

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***Abstract*— A double-Lorentz (DL) transmission line (TL) metamaterial is proposed for the first time. Both effective material parameters μ_{eff} and ε_{eff} of the corresponding line exhibit a Lorentz-type dispersion, hence the proposed terminology double-Lorentz. This type of TL presents the interest of being intrinsically tri-band, while the previously reported composite right/left-handed TL metamaterial is only dual-band. The DL TL theory is fully derived and demonstrated experimentally by a microstrip implementation. The tri-band property of the DL TL is illustrated by the design of a tri-band $\lambda/4$ impedance transformer and a systematic design procedure is described.**

***Index Terms*— Transmission line (TL) metamaterial, composite right/left-handed (CRLH) metamaterial, Drude and Lorentz media, tri-band components, impedance transformer.**

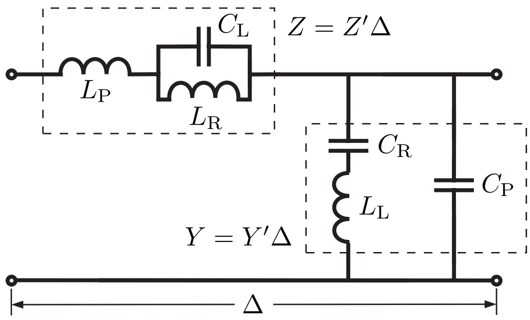

Figure 1: Incremental circuit model for the double-Lorentz (DL) transmission line (TL) metamaterial. This 1D model maybe extended to 2D and 3D real material configurations.

I. Introduction

Transmission line (TL) metamaterials have lead to many guided-wave and antenna applications in the microwave regime [1]. While the TL metamaterials proposed so far were essentially band-pass composite right/left-handed (CRLH) structures, dual stop-band CRLH metamaterials, with inverted left-handed and right-handed bands, have been reported recently [2]. However, such a dual CRLH structure is an idealization that cannot be exactly realized in practice. This paper shows that a real dual CRLH metamaterial is in fact a double-Lorentz (DL) medium and that this material exhibits an intrinsic tri-band property that may be exploited to design various tri-band transmission line based microwave components.

II. Double-Lorentz TL Metamaterial

Fig. 1 shows the incremental circuit model of a DL TL metamaterial, which consists in the combination of the ideal dual CRLH prototype [2] (parallel tank circuit with lumped elements $L_{\text{R}}, C_{\text{L}}$ in the series path of the TL and series circuit of $L_{\text{L}}, C_{\text{R}}$ in its shunt path) and of the parasitic series inductance L_{P} and shunt capacitance C_{P}. A DL TL does not exist in nature, but it may be engineered under the form of a metamaterial structure by repeating periodically the unit cell of Fig. 1, under the condition that this cell be subwavelength in the frequency range of operation. In this range, the response of the actual lumped-element structure is indiscernible from that of its idealized homogeneous counterpart. For this reason, we will in this section investigate the properties of the DL TL metamaterial mainly in the homogeneous limit, $\Delta/\lambda_{\text{g}} \to 0$, where the fundamental relations are particulary simple and insightful. The DL TL unit cell series impedance Z and shunt admittance Y (Fig. 1) are given by

$$Z(\omega) = Z'(\omega)\,\Delta = \mathrm{j}\,\omega L_{\text{P}} \frac{\omega^2 - \omega_0^{\text{se}\,2}}{\omega^2 - \omega_\infty^{\text{se}\,2}} = \mathrm{j}\,\omega\mu_0\mu_{\text{r}}(\omega)\,\Delta,$$

$$\text{with } \omega_\infty^{\text{se}} = \frac{1}{\sqrt{L_{\text{R}}\,C_{\text{L}}}} \text{ and } \omega_0^{\text{se}} = \sqrt{\frac{L_{\text{R}} + L_{\text{P}}}{L_{\text{R}} L_{\text{P}}\,C_{\text{L}}}}, \quad (1)$$

$$Y(\omega) = Y'(\omega)\,\Delta = \mathrm{j}\,\omega C_{\text{P}} \frac{\omega^2 - \omega_0^{\text{sh}\,2}}{\omega^2 - \omega_\infty^{\text{sh}\,2}} = \mathrm{j}\,\omega\epsilon_0\epsilon_{\text{r}}(\omega)\,\Delta,$$

$$\text{with } \omega_\infty^{\text{sh}} = \frac{1}{\sqrt{L_{\text{L}}\,C_{\text{R}}}} \text{ and } \omega_0^{\text{sh}} = \sqrt{\frac{C_{\text{R}} + C_{\text{P}}}{L_{\text{L}}\,C_{\text{R}} C_{\text{P}}}}. \quad (2)$$

The corresponding metamaterial constitutive parameters are directly obtained by Eqs. (1) and (2) as $\mu_{\text{r}}(\omega) = Z'(\omega)/(\mathrm{j}\omega\mu_0)$ and $\varepsilon_{\text{r}}(\omega) = Y'(\omega)/(\mathrm{j}\omega\varepsilon_0)$, respectively. These relations are plotted in Fig. 2 for a specific set of LC parameters. Both $\mu_{\text{r}}(\omega)$ and $\varepsilon_{\text{r}}(\omega)$ exhibit a Lorentz-type dispersion [3] – hence the proposed terminology double-Lorentz (DL) – whereas CRLH metamaterials exhibit a double-Drude response [1] and split-ring/thin-wire structures exhibit a mixed Lorentz-Drude response [4]. While a mixed Lorentz-Drude medium is fundamentally narrow

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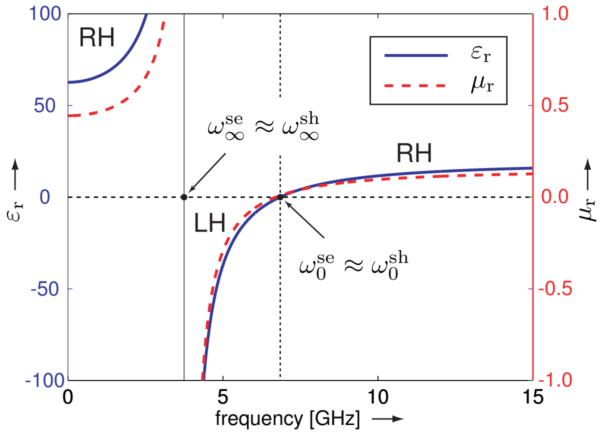

Figure 2: DL TL metamaterial constitutive parameters obtained from Eqs. (1) and (2) as $\mu_r(f) = Z'/(j2\pi f\mu_0)$ and $\varepsilon_r(f) = Y'/(j2\pi f\varepsilon_0)$ for the following parameters: $L_R = 1.30\,\text{nH}$, $C_L = 1.30\,\text{pF}$, $L_L = 1.33\,\text{nH}$, $C_R = 1.36\,\text{pF}$, $L_P = 0.65\,\text{nH}$, $C_P = 0.58\,\text{pF}$, $\Delta = 3.6$ mm.

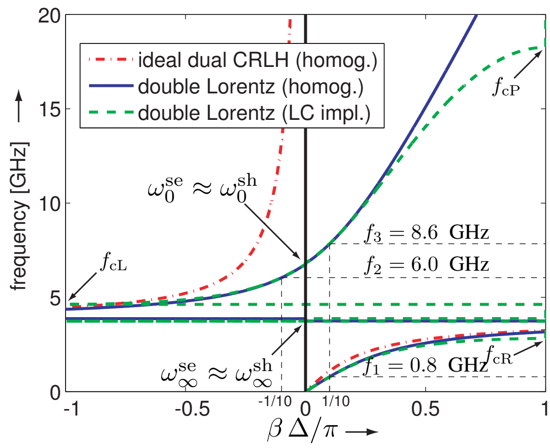

Figure 3: Dispersion diagram computed for the same parameter set as in Fig. 2 but different approximations are used: (1) ideal homogeneous dual CRLH TL [2], (2) homogeneous DL TL with the consideration of the parasitic elements L_P and C_P computed by Eq. (3) and (3) inhomogeneous DL TL where the finite length of the unit cell has been taken into account - plotted with Eq. (8).

band, due to the resonant nature of its Lorentz part, a DL medium can achieve a broader bandwidth, due to the *similar frequency dependence* of both μ and ε. Additionally, a DL medium has the useful property of being *tri-band* (Sec. IV), which can be exploited to transform microwave components into devices operating simultaneously at three different frequencies. By comparison, the Drude-Drude CRLH metamaterial is only dual-band.

The dispersion relation for the homogeneous DL TL, $k_h(\omega) = -j\gamma_h(\omega) = \beta_h(\omega) - j\alpha_h(\omega)$, is obtained from the immittances of Eqs. (1) and (2) as

$$k_h(\omega)\,\Delta = \sqrt{-Z(\omega)\,Y(\omega)} = \frac{\omega}{\omega_P}\sqrt{\frac{\omega^2-\omega_0^{se\,2}}{\omega^2-\omega_\infty^{se\,2}}\frac{\omega^2-\omega_0^{sh\,2}}{\omega^2-\omega_\infty^{sh\,2}}},$$

$$\text{with} \quad \omega_P = \frac{1}{\sqrt{L_P\,C_P}}, \tag{3}$$

and is plotted in Fig. 3 for the same parameters as in Fig. 2. As it may be seen in Eq. (3) and observed in Fig. 3, the DL TL medium has six characteristic eigenfrequencies, one zero at the origin $[\beta(\omega=0)=0]$, two poles delimiting a bandstop filter type gap $[\beta(\omega=\omega_\infty^{se},\omega_\infty^{sh})=\infty]$, two zeros $[\beta(\omega=\omega_0^{se},\omega_0^{sh})=0]$ higher in frequency delimiting a spectral gap, and one pole at high frequency corresponding to the low-pass filtering effect of the parasitic elements L_P and C_P. The characteristic impedance for the homogeneous DL TL, $Z_h(\omega)$, is obtained from the immittances of Eqs. (1) and (2) as

$$Z_h(\omega) = \sqrt{\frac{Z(\omega)}{Y(\omega)}} = \sqrt{\frac{L_P}{C_P}}\sqrt{\frac{\omega^2-\omega_0^{se\,2}}{\omega^2-\omega_\infty^{se\,2}}\frac{\omega^2-\omega_\infty^{sh\,2}}{\omega^2-\omega_0^{sh\,2}}}. \tag{4}$$

As the CRLH structure, the DL structure can be *balanced*, i.e. it can be designed as to exhibit a gap-less transition between the left-handed and right-handed bands. In the case of the DL medium this condition is twofold,

$$\omega_\infty^{se} = \omega_\infty^{sh} \equiv \omega_\infty \quad \text{and} \quad \omega_0^{se} = \omega_0^{sh} \equiv \omega_0. \tag{5}$$

In the design of Fig. 2, this condition is approximately satisfied. Under the balanced condition, the complex wavenumber, k_h in Eq. (3), and the complex characteristic impedance, Z_h in Eq. (4), reduce to purely real quantities,

$$\beta_h(\omega)\,\Delta = \frac{\omega}{\omega_P}\frac{\omega^2-\omega_0^2}{\omega^2-\omega_\infty^2}, \tag{6}$$

$$Z_h^{bal} = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{L_L}{C_L}} = \sqrt{\frac{L_P}{C_P}}, \tag{7}$$

and therefore the aforementioned medium gaps close up. Additionally, the characteristic impedance becomes frequency-independent, which enables broadband matching.

When the DL TL is implemented with real finite-size LC unit cells ($\Delta/\lambda_g \nrightarrow 0$) but still with subwavelength (or lumped) dimension ($\Delta/\lambda_g < 1/4$), all of the above analysis still holds, except that its accuracy progressively decreases as frequency moves away from $\omega = 0$ and $\omega = \omega_0$, which correspond to the phase origins ($\Delta/\lambda_g = 0$) of the DL structure. The corresponding dispersion relation is derived by Bloch theory [1] from the general formula $k(\omega)\Delta = \arccos[1 + Z(\omega)Y(\omega)/2]$, which becomes in the balanced case with Eqs. (1) and (2)

$$k(\omega)\,\Delta = \arccos\left[1 - \frac{1}{2}\left(\frac{\omega}{\omega_{\mathrm{P}}}\frac{\omega^2-\omega_0^2}{\omega^2-\omega_\infty^2}\right)^2\right]. \quad (8)$$

This relation shows the existence of two filtering stop bands (one stop-band ranges from f_{cR} to f_{cL} and the other one starts at f_{cP}) as depicted in Fig. 3. Exact formulas for the cutoff frequencies f_{cR}, f_{cL} and f_{cP} can be derived as solution of the equation $Z(\omega)\,Y(\omega) + 4 = 0$ [1], but this will not be done here due to lack of space.

After showing that the proposed TL metamaterial exhibits a DL phase dispersion, it is essential to investigate its impedance characteristics, more particularly its Bloch impedance, Z_{B}, which is the impedance that will be seen by the wave at the input of the DL TL and therefore the impedance that will determine matching. This impedance is identical to a homogeneous limit characteristic impedance [Eq. 7] at the phase origins [$Z_{\mathrm{B}}(\omega \to 0, \omega_0) \to \sqrt{L_{\mathrm{R}}/C_{\mathrm{R}}} = \sqrt{L_{\mathrm{L}}/C_{\mathrm{L}}}$] and may be set to the constant port termination at this frequencies, but varies both across the unit cell and in frequency away from the phase origins, which limits bandwidth. The Bloch impedance, computed from the ABCD matrix of the unit cell [1], is plotted in Fig. 4 to illustrate these facts.

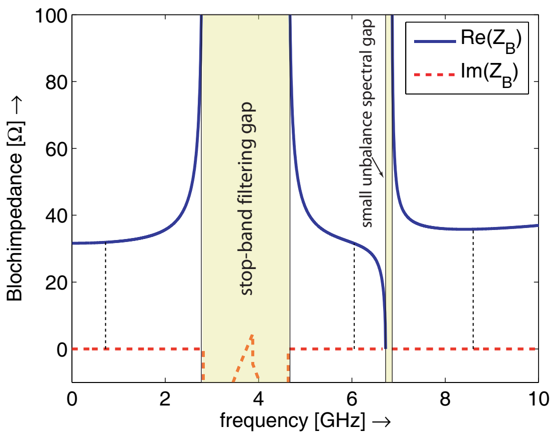

Figure 4: Bloch impedance $Z_{\mathrm{B}}(\omega)$ of the DL TL for the same parameters as in Fig. 2 (slightly unbalanced).

III. Practical Implementation

Figs. 5 and 6 show the perspective view of the unit cell layout and the prototype, respectively, of a DL TL structure implemented in microstrip technology. This structure is a two-metal layer (plus the ground plane) structure with short strip inductors and metal-insulator-metal (MIM) capacitors. In Fig. 7 the dispersion diagram for the design of Fig. 6, including measurement and full-wave (FDTD tool Empire XCcell) results using transmission phase unwrapping, $\beta\Delta = (-\varphi^{\mathrm{unwrapped}}\{S_{21}\} + m\cdot 2\pi)/N$ (where N is the number of cells and m is an integer selecting

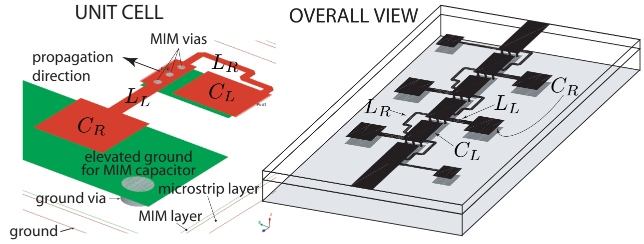

Figure 5: Microstrip implementation of a DL TL structure, including two metal layers in addition to the ground plane. The series inductance L_{P} and shunt capacitance C_{P} are of parasitic nature and thus implicit.

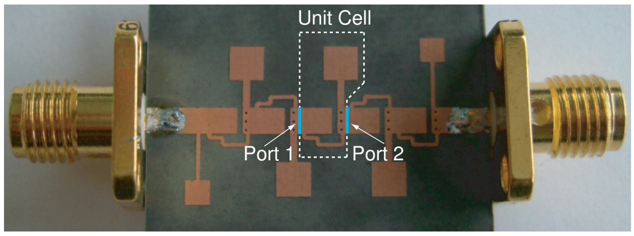

Figure 6: Microstrip DL TL prototype. Both substrates are Duroid 5870. The thicknesses of the lower and upper substrates are 0.508 mm and 0.127 mm, respectively. The structure includes $N = 5$ unit cells of length $\Delta = 3.6$ mm. The extracted DL LC parameters for this design are those given in the caption of Fig. 2.

the appropriate phase zero as the phase origin), as well as analytical results obtained by Eq. (8) from extracted LC parameters are depicted. Excellent agreement between these three types of results can be observed, which shows that the DL TL medium can be well realized in practice in planar circuit technology, analyzed by standard TL and Bloch theory, and perfectly modeled by the unit cell circuit of Fig. 1. At a phase origin the guided wavelength is infinite, as illustrated in the inset of Fig. 7. The scattering parameters for the microstrip DL TL of Fig. 6 are given in Fig. 8. The filtering stop band is clearly apparent. The insertion loss is relatively low in the pass bands, and could be further reduced by improving the balance (slightly imperfect here) of the design.

IV. Tri-Band Design Procedure and Application to a $\lambda/4$ Impedance Transformer

The DL TL is inherently *tri-band*, which means that is has sufficient degrees of freedom to achieve a given desired electrical length at three distinct frequencies while preserving matching. The TL has 6 variables, $L_{\mathrm{R}}, L_{\mathrm{L}}, L_{\mathrm{P}}, C_{\mathrm{R}}, C_{\mathrm{L}}, C_{\mathrm{P}}$, which are related to the

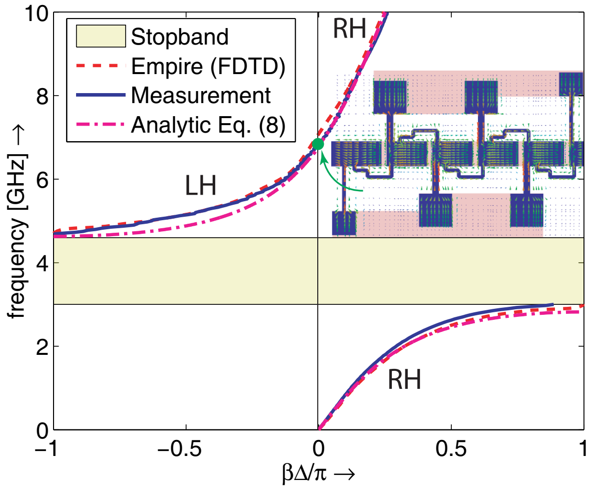

Figure 7: Dispersion diagram for the microstrip DL TL structure of Fig. 6, including measurement results, full-wave results (with the FDTD tool Empire XCcell) and analytical results obtained by Eq. (8). The inset shows the uniform field distribution at the phase origin.

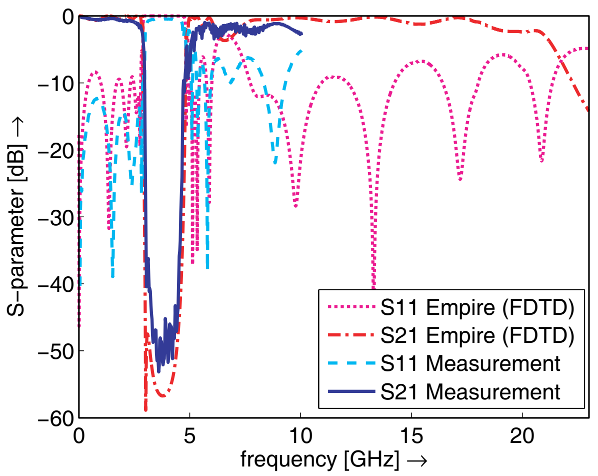

Figure 8: S-parameter for the microstrip DL TL of Fig. 6.

6 parameters, $\omega_\infty^{\mathrm{se}}, \omega_\infty^{\mathrm{sh}}, \omega_0^{\mathrm{se}}, \omega_0^{\mathrm{sh}}, \omega_{\mathrm{P}}$ and characteristic impedance. When the design is balanced [Eq. (5)], these parameters reduce to the 4 values $\omega_\infty, \omega_0, \omega_{\mathrm{P}}$ and $Z_{\mathrm{h}}^{\mathrm{bal}}$.

From these 4 parameters, the tri-band design procedure is as follows: 1) choose a TL characteristic impedance $Z_{\mathrm{h}}^{\mathrm{bal}}$, typically 50 Ω; 2) specify the three desired operating frequencies, ω_n $(n = 1, 2, 3)$, which will correspond to the intended TL phase shifts $\phi_n = -\beta_n N \Delta$, where the β_n's are obtained from Eq. (6); 3) select a number of cells N such that the phase shift of the cell at each frequency be smaller than 90° (metamaterial or longwavelength condition); 4) following Eq. (6), solve the system of 3 equations, $\phi_n \omega_{\mathrm{P}} \left(\omega_n^2 - \omega_\infty^2\right) + \omega_n \left(\omega_n^2 - \omega_0^2\right) = 0, \;\; (n = 1, 2, 3),$ in the three unknowns ω_{P}, ω_∞, ω_0; 5) design the layout of the DL TL unit cell that realizes the parameters $Z_{\mathrm{h}}^{\mathrm{bal}}$, ω_{P}, ω_∞, ω_0.

Fig. 9 shows the tri-band response of the DL TL in Fig. 6. This design is far from optimal (unbalanced) but it provides a proof of concept for the tri-band property of DL TL metamaterial structures.

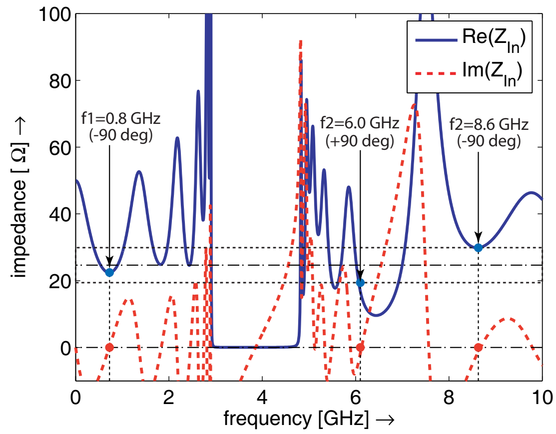

Figure 9: Input impedance $Z_{\mathrm{in}}(\omega)$ of the DL TL in Fig. 6 matching a load of $Z_{\mathrm{out}} = 50\ \Omega$ to $Z_{\mathrm{in}} = 22.5\,\Omega$ by providing the characteristic impedance of $Z_{\mathrm{transformer}} = \sqrt{Z_{\mathrm{in}} Z_{\mathrm{out}}} = 33.5\ \Omega$ and a phase shift of $\pm 90°$ at the three frequencies of operation ($f_1 = 0.8$ GHz, $f_2 = 6.0$ GHz and $f_3 = 8.6$ GHz).

V. Conclusion

A DL TL metamaterial has been presented for the first time. This DL TL metamaterial is intrinsically tri-band, while the CRLH version is only dual-band. The DL TL theory has been derived and a DL TL structure implemented in microstrip technology has been demonstrated experimentally. The tri-band property of the DL TL has been illustrated by the design of a tri-band $\lambda/4$ impedance transformer and a corresponding general design procedure has been described.

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