Extended Equivalent Circuit (EEC) FDTD: Special Computational Cells and their Stability Criteria

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Abstract — **An extended equivalent circuit (EEC) FDTD scheme is presented. This scheme exhibits high versatility, improved efficiency for the case of non-uniform meshes, and unconditional stability based on novel criteria for dispersive media and metamaterials. The EEC FDTD scheme applied to CRLH metamaterials may be regarded as the FDTD counterpart of the CRLH TLM scheme pioneered by Prof. W. J. R. Hoefer [1].**

The equivalent circuit (EC) FDTD scheme [2] is based on the *computational cell* shown in Fig. 1, which is the *circuital representation of the electromagnetic Yee cell*. This cell is obtained by spatially discretizing Maxwell's equations in their integral form and mapping them to Kirchoff's laws (Faraday → voltage law, Ampere → current law) with properly defined currents and voltages. The EC FDTD scheme is fundamentally a compact and efficient alternative representation of the conventional FDTD scheme and can be used in this initial form for the simulation of non-dispersive materials. Here the initial EC is extended in order to simulate dispersive media/metamaterials – hence the term EEC FDTD.

Firstly, the EEC FDTD scheme is *highly versatile*. Since its updated quantities are the voltages and currents, instead of the electric and magnetic field components, in the computational cell, the EEC FDTD allows *straightforward integration of electronic components* (inductors, capacitors, resistors, diodes, transistors, etc.) within the leap-frog algorithm and provides immediately the *circuital quantities of interest* for electronics systems and devices. Moreover, the EEC FDTD scheme *can be directly applied to analyze transmission line metamaterials* by insertion of the appropriate LC elements (additional shunt L and series C) in the computational cell [3].

Secondly, in the case of non-uniform meshes, and in particular meshes with abrupt variation in the sizes of neighboring cells (for instance typically required for narrow metallizations in microwave circuits), the EEC FDTD scheme exhibits a *larger maximum time step* (factor of 2 up to 10) *than that of the commonly used Courant-Friedrichs-Lewy (CFL) criterion* [4], allowing thereby faster computation. This benefit is specific to non-uniform meshes, while the CFL criterion is more favorable for uniform meshes (factor of $\sqrt{3}/2 \approx 0.87$ for 3D simulations). In practice, in the case of arbitrary structures with unpredictable meshes, the EEC and CFL criteria may be compared and the more advantageous of the two may be selected [5].

While the CFL criterion, which is based on non-dispersive media assumption, does not guarantee stability for dispersive media, the EEC FDTD scheme has been recently shown to provide *unconditional stability* both for non-dispersive and dispersive media (with different criteria), which represents an essential advantage for many media, such as for instance recent metamaterials. Two different approaches have been used to derive novel stability criteria for the EEC FDTD, one based on a Liapunov energy function [3, 4] and the other one based on an eigenvalue approach where the maximal eigenfrequency of the EC of the mesh limits the time step. These criteria may be applied for all types of dispersive media where an EC can be derived, including Drude/CRLH, Debye and Lorentz media or metamaterials.

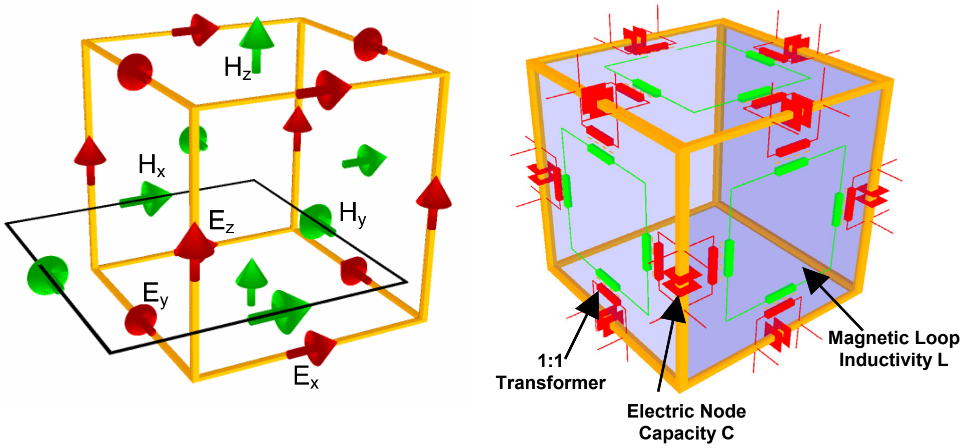

Fig. 1: Yee cell (left) and equivalent circuit (EC) based FDTD computational cell (right) for the simulation of non-dispersive material [5].

References

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