Tri-Band and Dual-Polarized Antenna Based on Composite Right/Left-Handed Transmission Line

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***Abstract*— Relaxing the composite right/left-handed (CRLH) balance condition enables arbitrary tri-band CRLH resonant devices. This paper presents a tri-band and dual-polarized antenna based on this principle. The proposed CRLH antenna operates in the transversally-polarized zeroth order (ZO) shunt mode ($n = 0$) and in the two longitudinally-polarized half-wavelength (HW) modes ($n = \pm 1$). Explicit synthesis formulas are given for the CRLH tri-band operation. A specific microstrip design using MIM (metal-insulator-metal) series capacitors and shunt stub inductors is demonstrated. In this design, the three resonant modes $n = -1$, $n = 0$ and $n = +1$ exhibited similar input impedances (allowing a simple feeding structure), radiation patterns and efficiencies/gains.**

I. INTRODUCTION

While conventional transmission lines (TLs) are single-band, *balanced* composite right/left-handed (CRLH) TL structures and devices are inherently *dual-band*, i.e. they may be designed to accomplish a given task at two different arbitrary frequencies [1]. This property results from the additional degree of freedom provided by a balanced CRLH TL compared to a conventional TL. As recalled in Fig. 1, a CRLH TL unit cell comprises the four parameters $\{L_\mathrm{R}, C_\mathrm{R}, L_\mathrm{L}, C_\mathrm{L}\}$, which, for a given structure period p, correspond to the dispersion relation $\beta(\omega) \approx \left[\omega\sqrt{L_\mathrm{R}C_\mathrm{R}} - 1/\left(\omega\sqrt{L_\mathrm{L}C_\mathrm{L}}\right)\right]/p$. Defining the left-handed (LH) and right-handed (RH) impedances $Z_\mathrm{L} = \sqrt{L_\mathrm{L}/C_\mathrm{L}}$ and $Z_\mathrm{R} = \sqrt{L_\mathrm{R}/C_\mathrm{R}}$, respectively, the balance condition, which corresponds to intrinsic matching of the TL (characteristic impedance Z_c = constant) and therefore broad bandwidth, provides a first design condition $Z_\mathrm{L} = Z_\mathrm{R} = Z_c$. Matching to constant port impedance Z_t (typically $Z_\mathrm{t} = 50$ Ω) gives the second condition $Z_\mathrm{c} = Z_\mathrm{t}$. At this point, there are only two equations for the four unknown parameters $\{L_\mathrm{R}, C_\mathrm{R}, L_\mathrm{L}, C_\mathrm{L}\}$; this provides the additional two degrees of freedom required for dual-band operation: $\beta(\omega_1) = \beta_1$ and $\beta(\omega_2) = \beta_2$, where $\{\omega_1, \omega_2\}$ are the center frequencies of the two operation bands and $\{\beta_1, \beta_2\}$ are the corresponding propagation constants yielding the desired response for the device.

From these explanations it appears that CRLH structures are restricted to dual-band operation. While this is true for broadband transmission line based devices, this paper shows that relaxing the balance condition allows *tri-band* operation in narrow-band or resonant-type applications, and in particular resonant antennas. This is a direct consequence of suppression of the condition $Z_\mathrm{L} = Z_\mathrm{R}$ from the balanced CRLH design, which yields the new system: $\{\beta(\omega_1) = \beta_1, \beta(\omega_2) = \beta_2, \beta(\omega_3) = \beta_3, Z_\mathrm{in} = Z_\mathrm{t}\}$. We present here a tri-band and dual-polarization CRLH antenna based on this principle. The design and optimization of this antenna have been carried out with the finite-difference time-domain (FDTD) software Empire XCcell and the method of moments (MoM) tool Ansoft Designer.

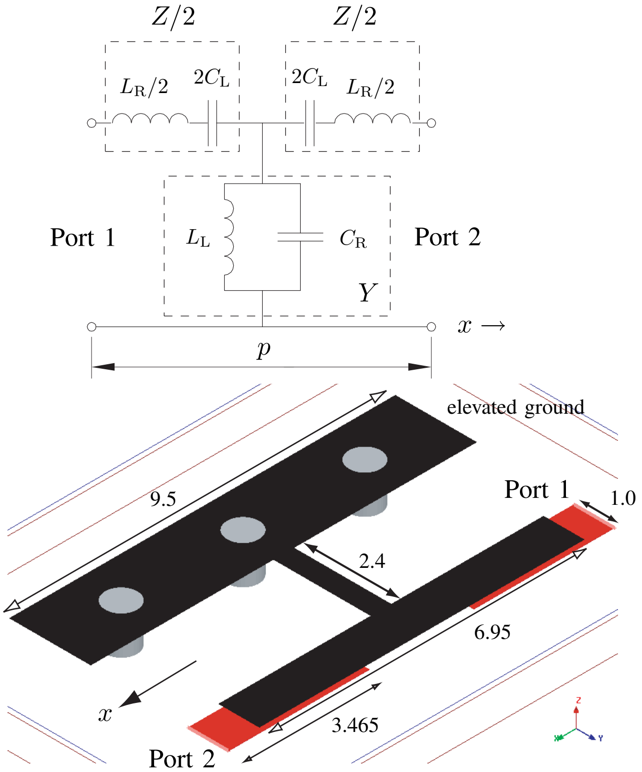

Fig. 1. CRLH unit cell topology along with its symmetric circuit model. The series C_L is implemented as MIM (metal-insulator-metal) capacitance and the shunt inductance L_L as short-circuited stub connected to an elevated ground plane. The substrate Rogers Duroid 5870 with height $h_1 = 127\,\mu$m (between the MIM plates) and $h_2 = 787\,\mu$m (between lower MIM plate and ground), $\varepsilon_r = 2.33$ and $\tan\delta = 1.2 \times 10^{-3}$ has been used. The layout parameters in mm of an optimized unit cell (for the tri-band antenna presented in section III) are also given. The width of the short-circuited stub measures 0.5 mm. The extracted parameters of the depicted unit cell are $L_\mathrm{R} = 3.06$ nH, $C_\mathrm{L} = 0.28$ pF, $L_\mathrm{L} = 0.88$ nH and $C_\mathrm{R} = 0.98$ pF.

II. General Tri-Band Design Procedure for an unbalanced CRLH Resonator

A CRLH TL is a periodic structure composed of cascaded unit cells. Fig. 1 shows a typical MIM (metal-insulator metal)/shorted-stub microstrip CRLH unit cell topology along with its lumped circuit model. Such an artificial TL exhibits the exact dispersion/attenuation relation [1]

$$\cos\left[(\beta - \mathrm{j}\alpha)\,p\right] = 1 - \frac{(\omega^2 - \omega_{\mathrm{se}}^2)(\omega^2 - \omega_{\mathrm{sh}}^2)}{2\,\omega^2\omega_{\mathrm{R}}^2}, \tag{1}$$

where $\omega_{\mathrm{se}}^2 = \frac{1}{L_{\mathrm{R}}C_{\mathrm{L}}}$, $\omega_{\mathrm{sh}}^2 = \frac{1}{L_{\mathrm{L}}C_{\mathrm{R}}}$, $\omega_{\mathrm{R}}^2 = \frac{1}{L_{\mathrm{R}}C_{\mathrm{R}}}$, so that the dispersion behavior of the line is described by only three independent eigenfrequencies. It will be shown below [Eq. (6)] that the fourth characteristic frequency $\omega_{\mathrm{L}}^2 = \frac{1}{L_{\mathrm{L}}C_{\mathrm{L}}}$ can be determined from the other three.

Such a CRLH dispersion diagram is plotted in Fig. 2 for a particular set of parameters. Due to the fact that the size p of the unit cell is strongly sub-wavelength, in the frequency range of operation ($p/\lambda_{\mathrm{g}} \ll 1$), CRLH structures behave as an uniform transmission media. Therefore all of the concepts of uniform traveling-wave and resonant-type structures can be transposed to such metamaterial structures, while they offer in addition unique properties not available in conventional systems.

Like any TL, a CRLH structure is transformed into a resonator when it is open-ended or short-ended. Due to its limited transmission band (Fig. 2), a CRLH resonator of N unit cells and length $\ell = Np$ exhibits $2N - 1$ resonance frequencies [1]. These frequencies are determined from the dispersion diagram by the condition $\ell = n\lambda_{\mathrm{g}}/2$, or equivalently, $\beta_n = n\pi/\ell = n\pi/(Np)$, with the CRLH particularity that n can be both positive and negative, and even zero.

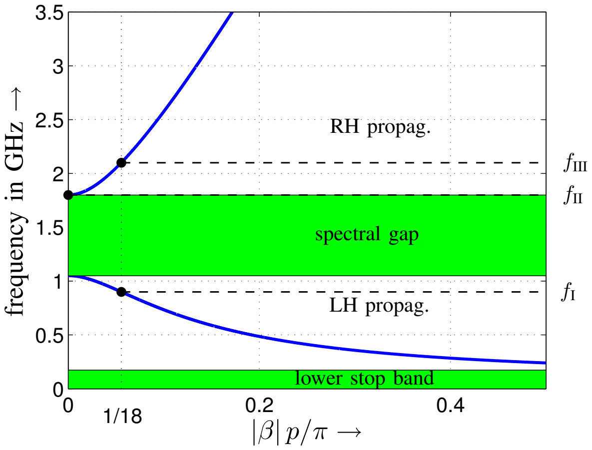

Fig. 2. Dispersion diagram [Eq. (1)] of an unbalanced CRLH TL with the resonances, $f_{\mathrm{I}} = f_{-1} = 0.9$ GHz, $f_{\mathrm{II}} = f_0 = f_{\mathrm{sh}} = 1.8$ GHz and $f_{\mathrm{III}} = f_{+1} = 2.1$ GHz indicating the proposed arbitrary tri-band design procedure. The CRLH parameters required for this tri-band design, obtained from the synthesis formulas of Eqs. (2), (4) and (5) with $N = 18$ are $L_{\mathrm{R}} = 0.74$ nH, $C_{\mathrm{L}} = 31.0$ pF, $L_{\mathrm{L}} = 6.6$ nH, and $C_{\mathrm{R}} = 1.2$ pF.

The half-wavelength (HW) regime uses the $n = \pm 1$ resonances and correspond to a quasi-sinusoidal series current distribution with a vanishing amplitude at the ends of the TL, yielding a broadside radiation with longitudinal (along the axis of the structure) polarization. Additionally, two zeroth order (ZO) modes ($n = 0$) are possible for a CRLH TL resonator [1]. If the TL is open-circuited at both ends, the so-called shunt mode with a spatially uniform shunt current (and theoretically zero series current), resulting in transverse (perpendicular to the axis of the structure) polarization is excited. This mode together with the two HW modes is used for the proposed tri-band antenna, as they were found to exhibit very close input impedances and similar absolute radiation pattern (due to the similar effective aperture) despite the different polarizations.

Let us now derive the synthesis formulas for the unbalanced CRLH tri-band design. The shunt ZO resonance frequency, corresponding to $\beta = 0$ or $\lambda_{\mathrm{g}} = \infty$, is immediately available from Eq. (1) as

$$\omega_{\mathrm{sh}}^2 = \frac{1}{L_{\mathrm{L}}C_{\mathrm{R}}} = \omega_{\mathrm{II}}^2. \tag{2}$$

At the two other operation frequencies, ω_{I} and ω_{III}, we will have $\beta\ell = \beta(Np) = \pm\pi$ or $\beta p = \pm\pi/N$ for HW regime. Insertion of these conditions into Eq. (1) yields the following 2-equations linear system

$$\begin{aligned}\omega_\nu^4 - \omega_{\mathrm{II}}^2\omega_\nu^2 - \omega_\nu^2\omega_{\mathrm{se}}^2 + \omega_{\mathrm{II}}^2\omega_{\mathrm{se}}^2 &= 2\,\omega_\nu^2\left[1 - \cos(\pi/N)\right]\omega_{\mathrm{R}}^2 \\ &= 4\,\omega_\nu^2 \sin^2\left[\pi/(2N)\right]\omega_{\mathrm{R}}^2, \quad \text{for} \quad \nu \in \{\mathrm{I}, \mathrm{III}\},\end{aligned} \tag{3}$$

from which the other two unknowns ω_{se}^2 and ω_{R}^2 may be determined as

$$\omega_{\mathrm{se}}^2 = \frac{1}{L_{\mathrm{R}}C_{\mathrm{L}}} = \frac{\omega_{\mathrm{I}}^2\omega_{\mathrm{III}}^2}{\omega_{\mathrm{II}}^2}, \tag{4}$$

$$\omega_{\mathrm{R}}^2 = \frac{1}{L_{\mathrm{R}}C_{\mathrm{R}}} = \frac{\omega_{\mathrm{I}}^2\omega_{\mathrm{II}}^2 + \omega_{\mathrm{II}}^2\omega_{\mathrm{III}}^2 - \omega_{\mathrm{I}}^2\omega_{\mathrm{III}}^2 - \omega_{\mathrm{II}}^4}{4\sin^2\left[\pi/(2N)\right]}. \tag{5}$$

When the three eigenfrequencies $\omega_{\mathrm{se}}, \omega_{\mathrm{sh}}$ and ω_{R} are known, the fourth dependent one, ω_{L}, can be calculated as follows

$$\omega_{\mathrm{L}}^2 = \frac{1}{L_{\mathrm{L}}C_{\mathrm{L}}} = \frac{\omega_{\mathrm{se}}^2\omega_{\mathrm{sh}}^2}{\omega_{\mathrm{R}}^2}. \tag{6}$$

Since at this point there have been only three linear independent eigenfrequencies, there is one degree of freedom left for the four LC parameters of the CRLH unit cell, which may be exploited to achieve a desired input impedance or to optimize other parameters such as gain or efficiency.

III. Tri-Band and Dual-Polarized CRLH Antenna with Broadside Fan-Beam Radiation

As shown in the previous section, the unbalanced CRLH design provides true tri-band operation. A balanced CRLH design, which may also be specified by $\omega_{\mathrm{se}} = \omega_{\mathrm{sh}} \equiv \omega_0$, is a particular case of the unbalanced one, where, according to Eqs. (2) and (4), the ZO frequency is equal to the geometric mean of the two HW frequencies: $\omega_{\mathrm{II}} = \sqrt{\omega_{\mathrm{I}}\omega_{\mathrm{III}}}$. For such a balanced design the two HW resonance frequencies can be determined from the general LC-based dispersion relation given in Eq. (1) by

$$\omega_{\mp 1} = \omega_{\mathrm{I,\,III}} = \sqrt{\omega_0^2 + \omega_{\mathrm{R}}^2 \sin^2(\pi/N)} \mp \omega_{\mathrm{R}} \sin(\pi/N). \tag{7}$$

We will use this particular case simply because this design was already available to us at the time of this writing. In the unbalanced case, the Bloch impedance of the TL, which corresponds to the true input impedance, will exhibit much stronger frequency variations. This fact may only restrict to some extent the operation bandwidth of each band of the TL resonantor but not invalidate the principle shown here. In the lower part of Fig. 1 the layout parameters of the proposed CRLH structure are given. Fig. 3 shows the current distributions corresponding to the three modes with the expected transverse ($n = 0$) and longitudinal ($n = \pm 1$) polarizations. These distributions clearly show a strong shunt current and correspondingly a small series current at the two ends of the antenna for all three modes, indicating a large input impedance, as verified in Fig. 4. A short 115Ω line together with a coupling MIM capacitor are used between the antenna and the exciting port in order to transform the high and relatively constant input impedance ($\approx 200\Omega$ for each of the three modes) to 50 Ω. Fig. 6 shows that good matching for all three frequency bands has been achieved with one feeding system. The 3D radiation patterns at the three frequencies, shown in Fig. 7 may be seen to be very close to each other. Finally, Figs. 8(a) to 8(c) show the 2D radiation patterns. In general, the co-to-cross polarization discrimination is very poor at off-broadside angles for all three modes. This may be understood by observing in Fig. 3 that the oppositely directed shunt currents fail to compensate toward grazing angles in the HW mode and non-negligible normal-to-ground-plane currents flow through the vias of the elevated ground in the ZO mode. The co-to-cross discrimination for the HW modes can be enhanced by alternating the stubs, but in this case, the ZO shunt mode is not an efficient radiator anymore due to current dipole moment cancellation.

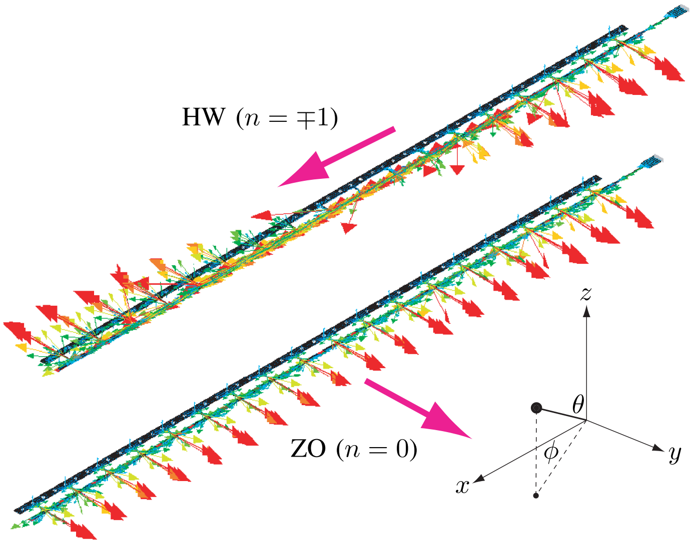

Fig. 3. Vectorial plot of the current densities $\vec{J}(x, y)$ for each mode. The large arrows indicate the overall polarization.

Tab. I shows comparative performances of the 3 bands. High directivities are achieved due to the very large effective

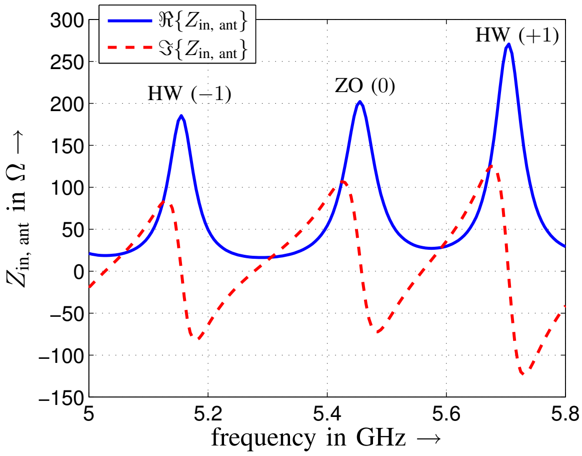

Fig. 4. Complex input impedance $Z_{\text{in, ant}}$ of the CRLH resonator (without matching network) indicating a similar real input impedance level of approx. 200Ω at the three modes $n \in \{-1, 0, +1\}$.

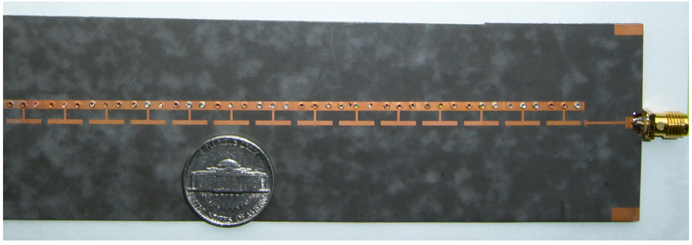

Fig. 5. Prototype of the CRLH antenna including matching network and SMA feed.

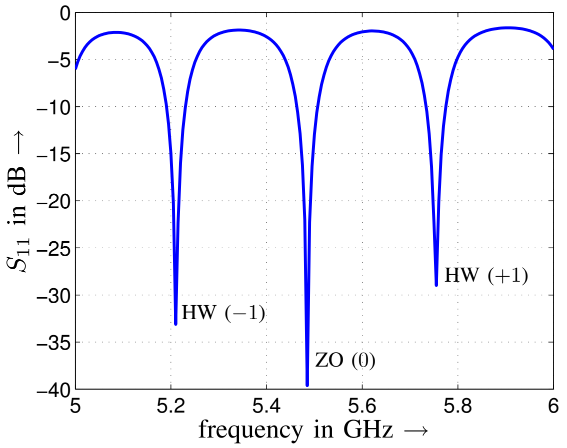

Fig. 6. Return loss of the tri-band CRLH antenna.

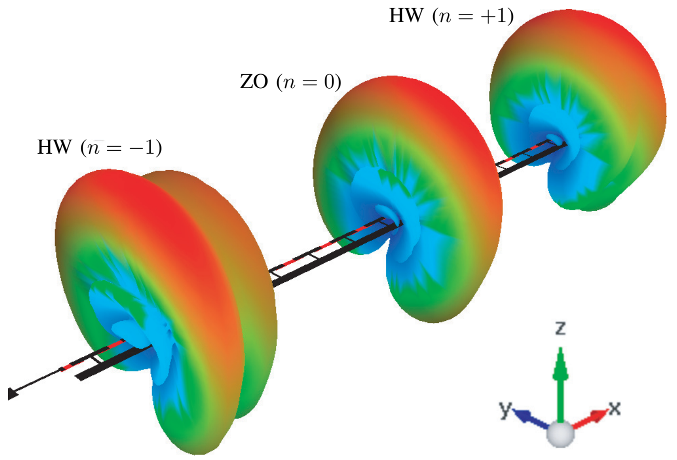

Fig. 7. 3D radiation pattern $E_{\text{abs}}(\theta, \phi)$ in dB for each of the three operation frequencies. The three plots indicate a similar absolute radiation characteristic.

apertures [2] obtained in such CRLH antennas, where the resonance frequencies are determined mainly by the LC loadings and not by the overall size, as first demonstrated in [3].

Mode n	-1	0	$+1$
f_{LC} in GHz [Eqs. (2) & (7)]	5.18	5.43	5.69
f_{FDTD} in GHz	5.21	5.49	5.76
length/λ_0	2.97	3.13	3.29
D in dBi	10.5	12.0	11.6
G in dB	9.0	10.5	10.1
Co-2-X pol. in dB	17.1	12.8	14.4
η_{rad} in %	71.1	70.9	71.0
HBW (E-/H-plane) in deg.	18/100	95/15	20/95

TABLE I
INTEGRAL PARAMETERS OF OPEN-ENDED CRLH ANTENNA IN EACH BAND OBTAINED BY FDTD SIMULATION.

IV. CONCLUSIONS

A novel general tri-band unbalanced CRLH operation principle has been demonstrated along with a resonant antenna application. The proposed CRLH antenna operates in the transversally-polarized ZO shunt mode and in the longitudinally-polarized two HW modes. The unique feature of this design is the similar input impedances, absolute radiation patterns and efficiencies of the three used modes.

ACKNOWLEDGMENT

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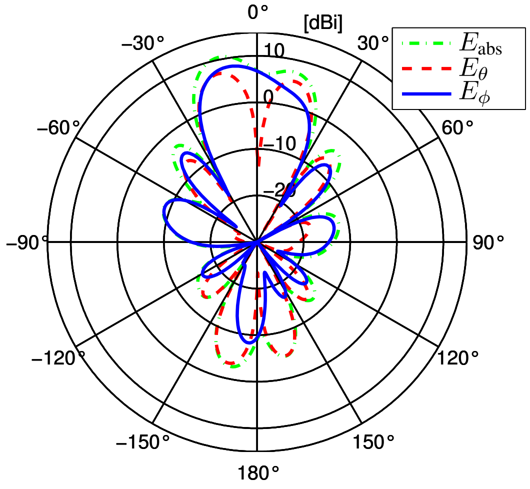
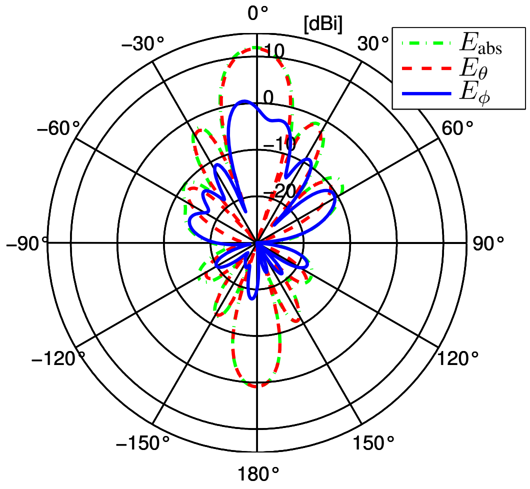
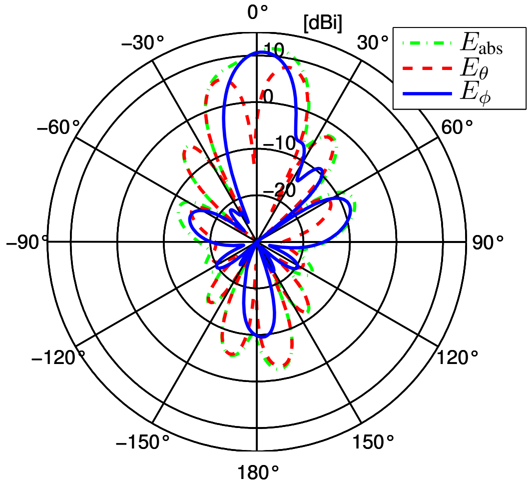


(a) left-handed half-wavelength mode



(b) zeroth order shunt mode



(c) right-handed half-wavelength mode

Fig. 8. FDTD-simulated co/cross-polarization and absolute directivity as a function of θ for the $\phi = 0°$ cut.