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Extended Composite Right/Left-Handed (E-CRLH) Metamaterial and its Application as Quadband Quarter-Wavelength Transmission Line

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***Abstract*— A novel extended composite right/left-handed (E-CRLH) transmission line (TL) metamaterial structure, constituted by the combination of the conventional CRLH (C-CRLH) and the recently introduced dual CRLH (D-CRLH) [1] prototypes, is proposed. This E-CRLH metamaterial is characterized by eight LC parameters (four C-CRLH and four D-CRLH parameters), which allow unprecedented diversity in the manipulation of the dispersion relation of the resulting TL structure. In particular, an E-CRLH TL metamaterial, under an extended balance condition, exhibits *two* frequencies of infinite wavelength propagation. In addition, the E-CRLH is intrinsically a *quadband* (arbitrary quadruplet of frequencies) structure. The latter property is exploited here into the design of a quadband quarter-wavelength transformer, and may be applied in principle to any TL-based microwave component.**

***Index Terms*— Metamaterials, composite right/left-handed (CRLH), dual CRLH (D-CRLH), extended CRLH (E-CRLH), quadband, quarter-wavelength transmission line.**

I. INTRODUCTION

Recently, emerging electromagnetic metamaterials have drawn considerable interest in the engineering and physics communities [2], [3]. A number of RF/microwave practical component, antenna and system applications, based on the powerful concept of composite right/left-handed (CRLH) metamaterials introduced in [4], have already been demonstrated to exhibit unprecedented performances and functionalities [2], [4].

One of the CRLH families of applications is that of dualband components [4], [5], enabled by the intrinsic four LC degrees of freedom (1 for balance, 1 for matching, and 2 for dual-band) of the conventional CRLH (C-CRLH) metamaterial prototype. However, no triband and even less quadband TL system has been reported to date. The present paper, by combining the C-CRLH and the dual CRLH (D-CRLH) introduced in [1] into an extended CRLH (E-CRLH) prototype, fills this gap, and paves the way for novel arbitrary tri- or quadband components. This may be of great benefit in modern multiband wired and wireless systems. The proposed E-CRLH TL metamaterial may be implemented either in chip or printed planar technologies.

II. E-CRLH TL METAMATERIAL

Fig. 1 shows the incremental circuit model of an E-CRLH TL metamaterial prototype, which consists in the combination of the C-CRLH and the D-CRLH prototypes. In the case of a hypothetical *homogeneous* ($\Delta/\lambda_g \to 0$) E-CRLH medium (non existing in nature but useful idealization, Sec. III), this prototype represents the infinitesimal equivalent circuit of the corresponding *uniform* TL, while in the case of a practical E-CRLH metamaterial ($0 < \Delta/\lambda_g \ll 1$) *lumped* implementation (Sec. IV), it represents the unit cell cascaded periodically to build the corresponding *effectively uniform* TL structure. The per-unit-length and times-unit-length LC parameters (denoted by primes) of the homogeneous idealization are related to the LC parameters (in H and F) of the lumped realization by: $L_\mathrm{R}'^{\mathrm{c,d}} = L_\mathrm{R}^{\mathrm{c,d}}/\Delta, C_\mathrm{R}'^{\mathrm{c,d}} = C_\mathrm{R}^{\mathrm{c,d}}/\Delta, L_\mathrm{L}'^{\mathrm{c,d}} = L_\mathrm{L}^{\mathrm{c,d}} \cdot \Delta$ and $C_\mathrm{L}'^{\mathrm{c,d}} = C_\mathrm{L}^{\mathrm{c,d}} \cdot \Delta$, where Δ represents the physical length (in m) occupied by the footprint of the lumped unit cell.

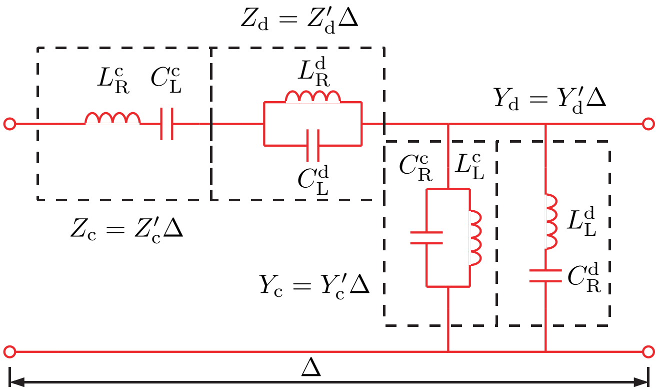

Figure 1: Incremental circuit model for the E-CRLH (extended) transmission line (TL) metamaterial. The superscript ”c” stands for C-CRLH (conventional) while the superscript ”d” stands for D-CRLH (dual). In the following, the E-CRLH will be designated by the superscript ”e”.

III. HOMOGENEOUS E-CRLH MEDIUM

While a homogeneous E-CRLH medium for a uniform E-CRLH TL does not exist in nature, it represents a simple and insightful idealization of its practical lumped-element implementation. In the frequency range of interest, $\Delta/\lambda_g \ll 1$, the response of the actual lumped-element structure is indiscernible from that of its idealized homogeneous counterpart. For this reason, we start to discuss the main properties of E-CRLH metamaterials in the homogeneous limit in this section.

The E-CRLH unit cell series impedance Z_e and shunt admittance Y_e (Fig. 1) are given by

$$Z_\mathrm{e} = Z'_\mathrm{e}\Delta = Z_\mathrm{c} + Z_\mathrm{d} = Z'_\mathrm{c}\Delta + Z'_\mathrm{d}\Delta, \quad \text{with} \tag{1a}$$

$$Z'_\mathrm{c} = \mathrm{j}\,\omega L'^\mathrm{c}_\mathrm{R}\left[1-\left(\frac{\omega^\mathrm{c}_\mathrm{se}}{\omega}\right)^2\right],\ \omega^\mathrm{c}_\mathrm{se} = \frac{1}{\sqrt{L'^\mathrm{c}_\mathrm{R}C'^\mathrm{c}_\mathrm{L}}}, \tag{1b}$$

$$Z'_\mathrm{d} = \frac{\mathrm{j}\,\omega L'^\mathrm{d}_\mathrm{R}}{1-(\omega/\omega^\mathrm{d}_\mathrm{se})^2},\ \omega^\mathrm{d}_\mathrm{se} = \frac{1}{\sqrt{L'^\mathrm{d}_\mathrm{R}C'^\mathrm{d}_\mathrm{L}}} \tag{1c}$$

and

$$Y_\mathrm{e} = Y'_\mathrm{e}\Delta = Y_\mathrm{c} + Y_\mathrm{d} = Y'_\mathrm{c}\Delta + Y'_\mathrm{d}\Delta, \quad \text{with} \tag{2a}$$

$$Y'_\mathrm{c} = \mathrm{j}\,\omega C'^\mathrm{c}_\mathrm{R}\left[1-\left(\frac{\omega^\mathrm{c}_\mathrm{sh}}{\omega}\right)^2\right],\ \omega^\mathrm{c}_\mathrm{sh} = \frac{1}{\sqrt{L'^\mathrm{c}_\mathrm{L}C'^\mathrm{c}_\mathrm{R}}} \tag{2b}$$

$$Y'_\mathrm{d} = \frac{\mathrm{j}\,\omega C'^\mathrm{d}_\mathrm{R}}{1-(\omega/\omega^\mathrm{d}_\mathrm{sh})^2},\ \omega^\mathrm{d}_\mathrm{sh} = \frac{1}{\sqrt{L'^\mathrm{d}_\mathrm{L}C'^\mathrm{d}_\mathrm{R}}}. \tag{2c}$$

The dispersion/attenuation relation $k(\omega) = -\mathrm{j}\gamma(\omega) = \beta(\omega) - \mathrm{j}\alpha(\omega)$ are then obtained from the above-mentioned per-unit-length immittances as

$$k_\mathrm{e} = \sqrt{-Z'_\mathrm{e}Y'_\mathrm{e}} = \sqrt{-Z'_\mathrm{c}Y'_\mathrm{c} - Z'_\mathrm{d}Y'_\mathrm{d} - Z'_\mathrm{c}Y'_\mathrm{d} - Z'_\mathrm{d}Y'_\mathrm{c}}. \tag{3}$$

The relations for the C-CRLH [2] and D-CRLH [1] represent the particular cases where $Z_\mathrm{d} = Y_\mathrm{d} = 0$ and $Z_\mathrm{c} = Y_\mathrm{c} = 0$, respectively, and are plotted in Fig. 2. The C-CRLH exhibits a low-frequency left-hand and a high-frequency right hand, while the opposite holds for the D-CRLH.

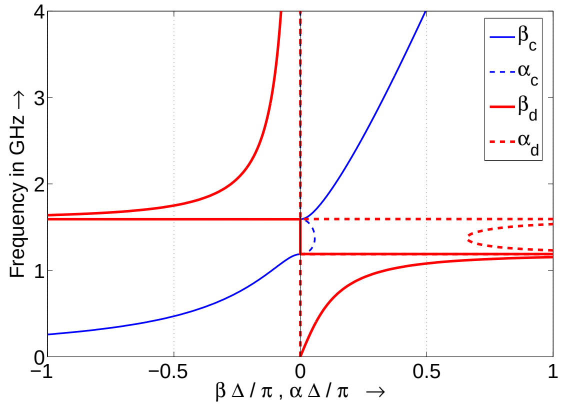

Figure 2: Unbalanced dispersion and attenuation diagram for the homogeneous C-CRLH and D-CRLH TLs.

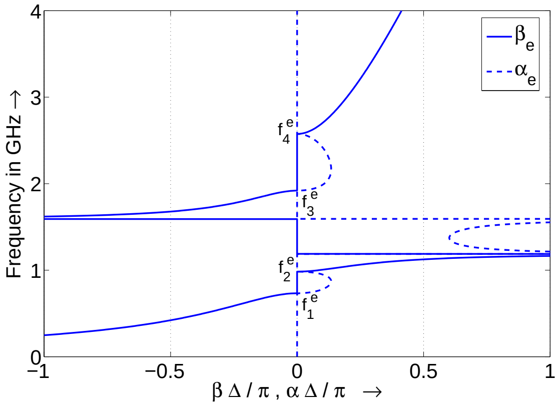

Figure 3: Unbalanced dispersion and attenuation diagram for the homogeneous E-CRLH TLs with the LC parameters $L'^\mathrm{c}_\mathrm{R} = 5$ nH/mm, $C'^\mathrm{c}_\mathrm{L} = 2$ pF·mm, $L'^\mathrm{d}_\mathrm{R} = 5$ nH/mm, $C'^\mathrm{d}_\mathrm{L} = 2$ pF·mm, $L'^\mathrm{c}_\mathrm{L} = 18$ nH·mm, $C'^\mathrm{c}_\mathrm{R} = 1$ pF/mm, L'^d_L =18 nH·mm, C'^d_R =1 pF/mm. At the four left-handed–right-handed transition frequencies $f^\mathrm{e}_{1,..,4}$, β_e vanishes.

The E-CRLH exhibits a richer behavior, with two left-handed bands and two right-handed bands, which is shown in Fig. 3. As the C-CRLH and the D-CRLH, the E-CRLH can be designed *balanced*, i.e. it can be designed to present gapless transmission and non-zero group velocities at the transition frequencies between the right-handed and left-handed bands. The E-CRLH balance condition is threefold:

$$\omega^\mathrm{c}_\mathrm{se} = \omega^\mathrm{c}_\mathrm{sh} = \omega^\mathrm{c}_0, \tag{4a}$$

$$\omega^\mathrm{d}_\mathrm{se} = \omega^\mathrm{d}_\mathrm{sh} = \omega^\mathrm{d}_0, \tag{4b}$$

$$\frac{L^\mathrm{d}_\mathrm{R}}{L^\mathrm{c}_\mathrm{R}} = \frac{C^\mathrm{d}_\mathrm{R}}{C^\mathrm{c}_\mathrm{R}} \quad \Leftrightarrow \quad \omega^\mathrm{c}_0 = \omega^\mathrm{d}_0 \equiv \omega_0, \tag{4c}$$

stating that the four resonance frequencies, $\omega^\mathrm{c}_\mathrm{se}, \omega^\mathrm{c}_\mathrm{sh}, \omega^\mathrm{d}_\mathrm{se}, \omega^\mathrm{d}_\mathrm{sh}$ are equal. If this condition is fulfilled, Eq. (3) reduces to the purely real expression

$$\beta_\mathrm{e} = \frac{\left(\omega^2 - \omega^{\mathrm{e}2}_1\right)\left(\omega^2 - \omega^{\mathrm{e}2}_2\right)}{\omega\,\omega'^\mathrm{c}_\mathrm{R}\,\left(\omega^2 - {\omega_0}^2\right)}, \tag{5}$$

with

$$\omega'^\mathrm{c}_\mathrm{R} = \frac{1}{\sqrt{L'^\mathrm{c}_\mathrm{R}C'^\mathrm{c}_\mathrm{R}}} \quad \text{in (rad·m)/s},$$

which is plotted in Fig. 4. Alternatively, Eq. (5) can be reformulated as

$$\beta_\mathrm{e} = \beta_\mathrm{c} + \beta_\mathrm{d} = \frac{\omega^2 - {\omega_0}^2}{\omega\,\omega'^\mathrm{c}_\mathrm{R}} - \frac{\omega\,\omega'^\mathrm{d}_\mathrm{L}}{\omega^2 - {\omega_0}^2}, \tag{6}$$

with the additional constant

$$\omega'^\mathrm{d}_\mathrm{L} = \frac{1}{\sqrt{L'^\mathrm{d}_\mathrm{L}C'^\mathrm{d}_\mathrm{L}}} \quad \text{in rad/(m·s)},$$

where the influence of the conventional and dual LC elements is separated into two superposed phase constants.

The two balanced transition frequencies $\omega^{\mathrm{e}}_{1,2}$ can be calculated by setting Eq. (6) to zero, which yields

$$\omega^{\mathrm{e}}_{1,2} = \sqrt{\omega_0^2 + \frac{\omega'^{\mathrm{c}}_{\mathrm{R}}\omega'^{\mathrm{d}}_{\mathrm{L}}}{4}} \mp \sqrt{\frac{\omega'^{\mathrm{c}}_{\mathrm{R}}\omega'^{\mathrm{d}}_{\mathrm{L}}}{4}} \tag{7}$$

and $\sqrt{\omega^{\mathrm{e}}_1\,\omega^{\mathrm{e}}_2} = \omega_0$.

Fig. 4 shows the balanced E-CRLH dispersion diagram. Most interestingly *two transition frequencies with non-zero group velocities*, allowing for instance, dual-band broadside leaky-wave radiation, are generated, while only one such frequency is available in the C-CRLH case. For the D-CRLH TL the phenomena of infinite wavelength propagation does not occur at all (without considering parasitic effects).

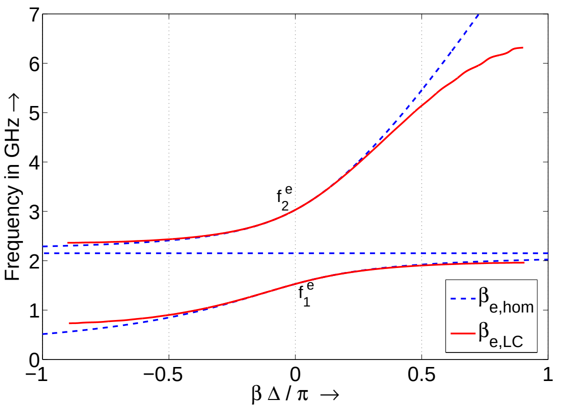

Figure 4: Balanced dispersion diagram for the homogeneous E-CRLH TL (dashed line) and its lumped-element implementation with 10 unit cells (of 1 mm length) for the LC parameters $L'^{\mathrm{c}}_{\mathrm{R}} = 3.04$ nH/mm, $C'^{\mathrm{c}}_{\mathrm{L}} = 1.8$ pF·mm, $L'^{\mathrm{d}}_{\mathrm{R}} = 1.47$ nH/mm, $C'^{\mathrm{d}}_{\mathrm{L}} = 3.71$ pF·mm, $L'^{\mathrm{c}}_{\mathrm{L}} = 4.5$ nH·mm, $C'^{\mathrm{c}}_{\mathrm{R}} = 1.21$ pF/mm, $L'^{\mathrm{d}}_{\mathrm{L}} = 9.27$ nH·mm, $C'^{\mathrm{d}}_{\mathrm{R}} = 0.59$ pF/mm.

The characteristic impedance of the E-CRLH TL is generally given by

$$Z^{\mathrm{e}}_0 = \sqrt{\frac{Z'_{\mathrm{e}}}{Y'_{\mathrm{e}}}} = \sqrt{\frac{L'^{\mathrm{c}}_{\mathrm{R}}}{C'^{\mathrm{c}}_{\mathrm{R}}}} \sqrt{\frac{1-(\omega^{\mathrm{c}}_{\mathrm{se}}/\omega)^2 + \frac{L'^{\mathrm{d}}_{\mathrm{R}}/L'^{\mathrm{c}}_{\mathrm{R}}}{1-(\omega/\omega^{\mathrm{d}}_{\mathrm{se}})^2}}{1-(\omega^{\mathrm{c}}_{\mathrm{sh}}/\omega)^2 + \frac{C'^{\mathrm{d}}_{\mathrm{R}}/C'^{\mathrm{c}}_{\mathrm{R}}}{1-(\omega/\omega^{\mathrm{d}}_{\mathrm{sh}})^2}}}. \tag{8}$$

In the balanced case [Eq. 4], it reduces to the purely real and frequency independent – and therefore broadband – expression

$$Z^{\mathrm{e,bal}}_0 = \sqrt{\frac{L'^{\mathrm{c}}_{\mathrm{R}}}{C'^{\mathrm{c}}_{\mathrm{R}}}} = \sqrt{\frac{L'^{\mathrm{c}}_{\mathrm{L}}}{C'^{\mathrm{c}}_{\mathrm{L}}}} = \sqrt{\frac{L'^{\mathrm{d}}_{\mathrm{R}}}{C'^{\mathrm{d}}_{\mathrm{R}}}} = \sqrt{\frac{L'^{\mathrm{d}}_{\mathrm{L}}}{C'^{\mathrm{d}}_{\mathrm{L}}}}. \tag{9}$$

IV. LUMPED-ELEMENT IMPLEMENTATION

The ideal-homogeneous E-CRLH of the previous section can be implemented under the from of a LC-network with the *symmetric* [2] unit-cell shown in Fig. 5. Due to the presence of real inductors and capacitors, this implementation exhibits filtering *resonances or Bragg gaps* in addition to the the *medium gaps* already present in the unbalanced homogeneous case (Fig. 3). For simplicity, we will consider here only the balanced case, which is the most useful in practice.

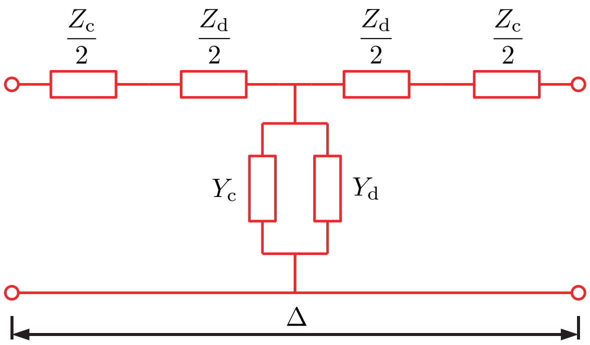

Figure 5: Symmetric T-network model of the E-CRLH unit cell

The S-parameters, plotted in Fig. 6, show the presence of a low-frequency (from DC) and a high-frequency gap (to ∞), due to the band-pass nature of the C-CRLH elements, as well as an unavoidable intermediate gap, due to the band-stop nature of the D-CRLH elements [1]. Fig. 7 shows the unwrapped phase of S_{21}.

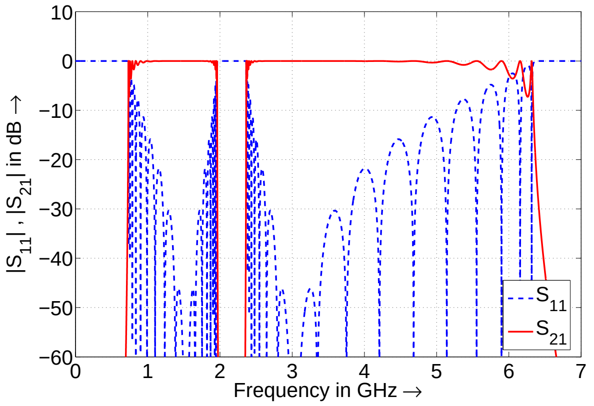

Figure 6: S-Parameter of a balanced E-CRLH TL consisting of 10 T-symmetric unit-cells for the same parameters as in Fig. 4 with $\Delta = 1$ mm.

The unwrapped phase of the transmission parameter $\phi^{\mathrm{unwrapped}}[S_{21}]$ depicted in Fig. 7 has been used together with the two calculated transition frequencies (Eq. 7) to extract the dispersion behavior of the $N = 10$ cell TL ($\beta_e\,\Delta = -\phi^{\mathrm{unwrapped}}/N + \xi$). This dispersion function

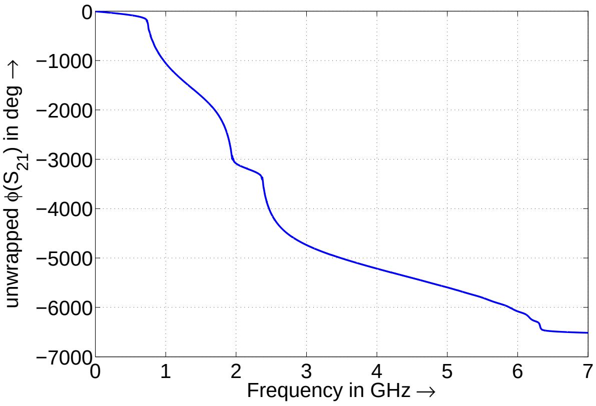

Figure 7: Unwrapped phase of S_{12} of an E-CRLH TL consisting of ten unit cells in symmetric T-form

is shown by the solid curve in Fig. 4, where the expected perfect agreement with the ideal E-CRLH is clearly visible in the vicinity of the two balanced transition frequencies (where $\lambda_{\mathrm{e}} = 2\pi/\beta_{\mathrm{e}} = \infty$ and therefore $\Delta/\lambda_{\mathrm{g}} = 0$).

V. QUADBAND QUARTER-WAVELENGTH TRANSMISSION LINE AND STUBS

The E-CRLH comprises twice the number of parameters (8 as opposed to 4) compared to the conventional and dual CRLH cases and these additional degrees of freedom lead to an intrinsic *quadband* property. In the balance case, there are 3 conditions for balance [Eq. 4] and one additional condition for matching to external ports [$Z_0^{\mathrm{e}} = 50\ \Omega$ in Eq. 9]. This leaves out four degrees of freedom, that may be readily exploited to generate the arbitrary-frequencies quadruplet $[\beta(\omega_1), \beta(\omega_2), \beta(\omega_3), \beta(\omega_4)] = [\beta_1, \beta_2, \beta_3, \beta_3]$, i.e. the quadband operation. The corresponding formulas, given the 8 LC parameters as a function of the four frequencies, four $\beta' s$, termination impedance and number of cells are straightforward but lengthy; due to lack of space, they will be given elsewhere.

For the sake of illustration, an E-CRLH quadband open-circuited stub, represented in Fig. 8, is presented here. A demonstration of the principle is shown in Fig. 9 by plotting the scattering parameters of the two-port.

VI. CONCLUSION

A novel extended composite right/left-handed (E-CRLH) transmission line (TL) metamaterial structure has been proposed. This metamaterial has been shown to exhibit two infinite wavelength propagation frequencies between the left-handed and right-handed bands and to provide arbitrary quadband operation. As an illustration of the latter property, a quadband quarter-wavelength TL has been demonstrated. Due to the richness of its dispersion relation, this E-CRLH is expected to lead to numerous microwave applications.

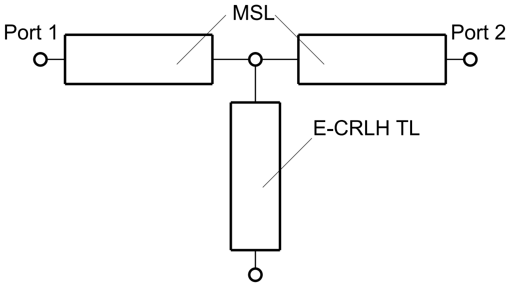

Figure 8: Microstrip line with quadband open-circuited E-CRLH TL stub at its center.

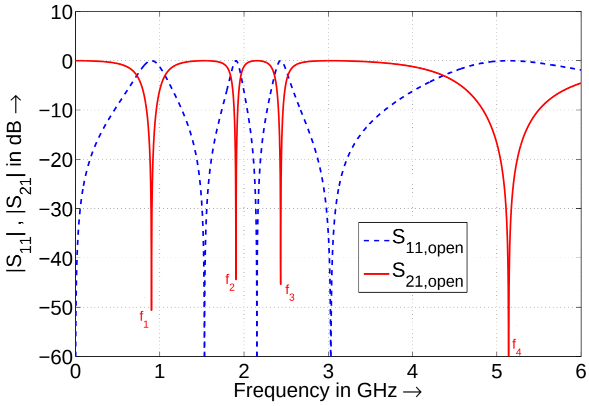

Figure 9: Simulated S-Parameter of quadband open-circuited stub with the four operation frequencies $f_1 =$900 MHz, $f_2 = 1900$ MHz, $f_3 = 2.430$ GHz and $f_4 = 5.140$ GHz with the choice of the phases $\beta_{\mathrm{e}}(f_1)\,\Delta = \beta_{\mathrm{e}}(f_3)\,\Delta = -\pi/2, \beta_{\mathrm{e}}(f_2)\,\Delta = \beta_{\mathrm{e}}(f_4)\,\Delta = +\pi/2$. The resulting parameters are $L_{\mathrm{R}}^{\mathrm{c}} = 3.04$ nH, $C_{\mathrm{L}}^{\mathrm{c}} = 1.8$ pF, $L_{\mathrm{R}}^{\mathrm{d}} = 1.47$ nH, $C_{\mathrm{L}}^{\mathrm{d}} =$3.71 pF, $L_{\mathrm{L}}^{\mathrm{c}} =$4.5 nH, $C_{\mathrm{R}}^{\mathrm{c}} =$1.21 pF, $L_{\mathrm{L}}^{\mathrm{d}} =$9.27 nH, $C_{\mathrm{R}}^{\mathrm{d}} = 0.59$ pF.

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